A Tentative Completion of Quantum Mechanics[1]

"One is [thus] led to conclude that the description of reality as given by a wave function is not complete." (A. Einstein)

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Contents and credits

Contents:

Credits:

I wish to thank my collaborators on projects related to this lecture, in particular my last PhD student *Baptiste Schubnel* and my friend *Alessandro Pizzo*, for the joy of our joint efforts, and numerous colleagues in many places, including *Detlev Buchholz,* for useful discussions of various aspects of **Quantum Mechanics**.

Apologies:

I won't be able to describe the nitty-gritty details of my approach to *Relativistic Quantum Theory.*

Summary

I describe some ideas about how to complete (non-relativistic) *Quantum Mechanics (QM)* to a theory that makes sense. My proposal is called

ETH - **Approach** to *QM*

"*E*" standing for Events, "*T*" for Trees, and "*H*" for Histories. This approach supplies the last one of **three pillars** *QM* can be constructed upon, which are:

(i) Physical quantities characteristic of a system are represented by selfadjoint operators. Their time evolution is given by the *Heisenberg equations*.

(ii) Appropriate notions of *states* and of *potential* and *actual events*.

(iii) A general *Law for the Time Evolution of States* of a system.

Besides providing some details on pillars (ii) and (iii), I will sketch how the *ETH*-Approach to *QM* can be reconciled with *Relativity Theory*.

My general goal is to help removing some of the **enormous confusion** befuddling many people who claim to work on the foundations of *QM*.

1. What's missing in text-book *QM*?

"It seems clear that the present quantum mechanics is not in its final form." (Paul Adrien Maurice Dirac)

Text-book Quantum Mechanics is a theory of *ensembles* of identical physical systems and of their time evolution – alas, incomplete – based on the following two pillars:

(i) A system, S, is characterized by a list

$$\mathcal{O}_S = \left\{ \widehat{X}_\iota = \widehat{X}_\iota^* \middle| \iota \in \mathfrak{I}_S \right\}$$

of abstract bounded self-adjoint operators representing *physical quantities* characteristic of S, such as the electromagnetic field, the total momentum, energy or spin of all particles in S ... localized in a bounded region of space-time. In general, different operators in $\mathcal{O}_S$ do *not* commute with one another. One assumes that if $\widehat{X} \in \mathcal{O}_S$ and F is a real-valued, bounded continuous function on $\mathbb{R}$ then $F(\widehat{X}) \in \mathcal{O}_S$, too.

In general, $\mathcal{O}_S$ does not have any additional structure – it is usually not a real linear space, let alone an algebra.

The two pillars text-book *QM* is based upon

At every time t, there is a representation of $\mathcal{O}_S$ by bounded self-adjoint operators acting on a separable Hilbert space $\mathcal{H}$:

$$\mathcal{O}_S \ni \widehat{X} \mapsto X(t) = X(t)^* \in B(\mathcal{H}) \tag{1}$$

<u>*Heisenberg picture:*</u> If S is an **isolated** system then the operators $X(t)$ and $X(t')$ representing a physical quantity $\widehat{X} \in \mathcal{O}_S$ at two arbitrary times t and t' are unitarily conjugated to one another,

$$X(t') = U_S(t',t)^* \, X(t) \, U_S(t',t), \tag{2}$$

where $\left\{ U_S(t',t) \middle| t,t' \in \mathbb{R} \right\}$ is the unitary propagator of S. The system S is *autonomous* iff $U_S(t',t) = \exp[-i(t'-t)H_S/\hbar]$, for arbitrary t and t', where H_S is the Hamiltonian of S.

(ii) "States" of S are given by density matrices Ω. The expectation at time t of an operator $\widehat{X} \in \mathcal{O}_S$ in the "state" Ω of S is given by

$$\omega\big(X(t)\big) := \mathrm{Tr}\big(\Omega \, X(t)\big).$$

Ω is pure iff it is given by a rank-1 orgthogonal projection π.

The "blunder" of the Schrödinger picture and -equation

In text-book *QM*, it is usually assumed (following *Schrödinger*) that, in the *Heisenberg picture*, "states" of an isolated physical system are *independent* of time t, and, hence, that the Heisenberg picture is equivalent to the *Schrödinger picture*:

$$\omega(X(t)) = \mathrm{Tr}\big(\Omega\, X(t)\big) = \mathrm{Tr}\big(\Omega(t)\, X\big), \quad X := X(t_0),\, \Omega := \Omega(t_0),$$

where the time-dependence of $\Omega(t)$ is determined by the *Schrödinger-von Neumann* equation

$$\dot{\Omega}(t) = -\frac{i}{\hbar}\big[H(t), \Omega(t)\big]\,. \tag{3}$$

[More generally, the time-dependence of states of a system S interacting with its environment is described by *completely positive linear* maps, $\{\Gamma(t,t')\,|\,t \geq t'\}$, with $\Gamma(t,t') = \Gamma(t,t'') \cdot \Gamma(t'',t')$ and $\Gamma(t,t) = \mathbf{1}$:

$$\Omega(t) = \Gamma(t,t')\big[\Omega(t')\big]\,, \qquad \forall t' \geq t\,. \quad] \tag{4}$$

One thus observes that, in text-book *QM*, the *time evolution of states* in the Schrödinger picture (see Eqs. (3), (4)) is **linear** and **deterministic**!

But what about the probabilistic nature of *QM*?

Of course, this **cannot be the full story**, as already discovered by *Einstein,* in 1916, in his analysis of spontaneous and induced emission and absorption of photons by atoms (*fluorescence*; A - and B - coefficients).

According to the *Copenhagen interpretation* of *QM*, the deterministic evolution (3) of the state of S is *"interrupted"* at all times t when an *"event"* happens, such as the emission or absorption of a photon by an atom, or the completion of a measurement of the value of a physical quantity $\widehat{X} \in \mathcal{O}_S$, in which case the "state" of S exhibits a *"quantum jump"* to a state in the range of the spectral projection of $X(t)$ corresp.to the eigenvalue of $X(t)$ that equals the value of $\widehat{X}$ measured at time t. *QM* is claimed to give the **probabilities** of quantum jumps to different eigenstates of $X(t)$: These probabilities are given by *Born's Rule*.

If the machinery used to measure the value of $\widehat{X}$ is *included* in what constitutes the *total* system S (now assumed to be *isolated*) one might expect, erroneously (!), that the event corresponding to a measurement of the value of $\widehat{X}$ could be viewed as the result of the *Schrödinger-Liouville evolution of the state of the* **total** *system*. This would imply that *QM* is deterministic – *which it obviously isn't!* So, what's going on?

Analogy with Brownian motion

Well, the qm evolution of an **average** of states of very many **identical** isolated physical systems appears to be correctly described by the Schrödinger-von Neumann evolution; but the qm evolution of the state of an **individual** system is **stochastic**.

Thus, what's missing in text books is an answer to the *fundamental question*: *What is a completion of QM that correctly describes the stochastic evolution of the state of an individual isolated physical system?*

A useful analogy: Consider a system consisting of very many identical "test particles" suspended in a liquid and exhibiting *Brownian motion*. Let $\rho_t(x), x \in \mathbb{R}^3$, be the the "state" of the system at time t, namely the normalized particle density. Its time-dependence is governed by the **diffusion equation**, which is a *deterministic, linear* law of evolution:

$$\dot{\rho}_t(x) = D\,(\Delta\rho_t)(x), \qquad D : \text{diffusion constant.} \tag{5}$$

Solution:

$$\rho_t(x) = \int_{\mathbb{R}^3} d^3x'\, \Gamma_{t-t'}(x-x')\rho_{t'}(x'), \quad \Gamma_t(x) := (2\pi D t)^{-\frac{3}{2}}\, e^{-\frac{|x|^2}{2Dt}}\,,$$

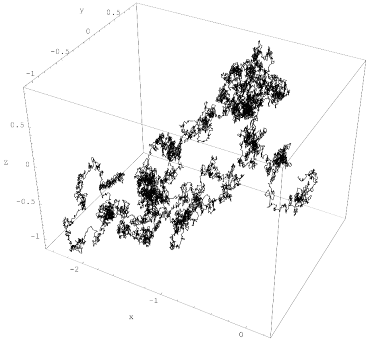
The heat kernels Γ_t satisfy the *Chapman-Kolmogorov* equation.

Ontology of Brownian motion according to Einstein, Smoluchowski and Wiener

Ontology: Consider a system consisting of a single test particle suspended in a fluid. Its Brownian motion arises from its random collisions with compounds of fluid molecules ($\Rightarrow D = \frac{k_B T}{6\pi\eta r}, \ldots$); and

(i) the test particle is localized in a point $x_\xi(t) \in \mathbb{R}^3$ at every time t;

(ii) its trajectory $\xi := \left\{x_\xi(t)\right\}_{t \geq t_0}$ is a random continuous curve in physical space $\mathbb{R}^3$, as indicated in the figure below.



A measure on the "space of histories"

As shown by *N. Wiener*, $\exists$ a *probability measure*, $dW_{x_0}(\xi)$, on the space, Ξ, of particle trajectories, $\xi\big(:= \big\{x_\xi(t) \in \mathbb{R}^3 \big| t \geq t_0, x_\xi(t_0) = x_0\big\}\big)$, that start at x_0 at time t_0; dW_{x_0} is supported on trajectories ξ that are *Hölder continuous of index* $\frac{1}{2}$, etc.

An *"event"* at time t is the sight of the test particle at a pt. $x_\xi(t) \in \mathbb{R}^3$. The trajectory ξ can thus be viewed as a *"history of events"* – a **random object** – and Ξ is the *"space of histories."* Wiener measure allows us to predict probabilities of *measurable sets of histories*. Example:

$$\text{prob}\big\{\xi \in \Xi \,\big|\, x_\xi(t_i) \in \mathcal{O}_i,\ i = 1, 2, \dots, n,\ t_0 < t_1 < \cdots < t_n\big\}$$
$$= \int_\Xi dW_{x_0}(\xi) \prod_{i=1}^{n} \chi\big(\xi \big| x_\xi(t_i) \in \mathcal{O}_i\big)\,. \tag{6}$$

Using Wiener measure to take an **average** over an ensemble of *very many identical systems of test particles*, one recovers the *deterministic linear law* in Eq. (5) for the evolution of the "state" ρ_t:

$$\rho_t(x) = \int d^3x_0\, \Gamma_{t-t_0}(x - x_0)\rho_{t_0}(x_0)$$
$$= \int d^3x_0\, \rho_{t_0}(x_0) \int_\Xi dW_{x_0}(\xi)\, \chi\big(\xi \big| x_\xi(t) = x\big). \tag{7}$$

2. "Unraveling" the Schrödinger-Liouville equation

"If you are receptive and humble, mathematics will lead you by the hand." (P.A.M. Dirac)

Chapman-Kolmogorov for $\Gamma_t \Rightarrow$ Markov property for dW_{x_0}. One says that the Wiener measure *"unravels"* the diffusion equation (5).

In the following I propose a completion of *QM* involving an *"unraveling"* of the Schrödinger-Liouville equation for the propagation of "states" of individual systems. But it won't be Markovian!
Upshot of analysis: The Schrödinger-Liouville equation describes the evolution of the state *averaged over the random histories of many identical, identically prepared systems*.

The **ontology of** *QM* lies in *"random histories* of *events"*; and *QM* equips the (non-commutative) space of such histories with a *"quantum probability measure"*, in analogy with the Wiener measure for Brownian motion. Our task is to *find* this probability measure, or, put differently, *to find an appropriate notion of states of physical systems and to describe their stochastic time evolution.* – The *ETH - Approach to QM*, developed during the past decade, accomplishes this task! It will be sketched for non-relativistic *QM*; (but there exists a *relativistic version* of it).

Elements of the *ETH* - Completion of no-relat. *QM*

It is convenient to introduce some natural algebras.

$$\mathcal{E}_I := \langle X(t) \big| \widehat{X} \in \mathcal{O}_S, t \in I \subset \mathbb{R} \rangle, \qquad \mathcal{E}_{\geq t} := \overline{\bigvee_{I \subset [t,\infty)} \mathcal{E}_I}, \qquad (8)$$

[where we take a closure in the weak topology of $B(\mathcal{H})$, ...]. Clearly

$$\mathcal{E}_{\geq t} \supseteq \mathcal{E}_{\geq t'}, \quad \text{whenever} \;\; t' > t\,.$$

In text-book examples, $\mathcal{E}_{\geq t}$ is *indep.* of time t. $\rightarrow$ Measurement Problem!

Definition 1: Let S be an **isolated** (...) physical system. **Potential (future) events**, $\mathfrak{e}$, in S (*"potentialities"*) at times $\geq t$ are described by partitions of unity,

$$\mathfrak{e} = \{\pi_\xi \,|\, \xi \in \mathfrak{X}_\mathfrak{e}\} \subset \mathcal{E}_{\geq t}, \quad \mathfrak{X}_\mathfrak{e} \; \text{countable}\,, \sum_{\xi \in \mathfrak{X}_\mathfrak{e}} \pi_\xi = \mathbf{1}\,, \qquad (9)$$

by *disjoint orthogonal projections*, $\pi_\xi = \pi_\xi^* \in \mathcal{E}_{\geq t}$.

In the following, the symbol π will always stand for an orthogonal projection, $\pi = \pi^* = \pi^2$, acting on $\mathcal{H}$.

The Principle of Diminishing Potentialities

"Indeed, it is evident that the mere passage of time itself is destructive rather than generative ..., because change is primarily a 'passing away."'

(Aristotle, Physics)

An *isolated* system S is characterized by a co-filtration, $\{\mathcal{E}_{\geq t} \,|\, t \in \mathbb{R}\}$, of *algebras of potential future events; S* is *autonomous* iff

$$\mathcal{E}_{\geq t'} = e^{i(t'-t)H_S}\,\mathcal{E}_{\geq t}\,e^{i(t-t')H_S}\,, \quad (H_S \text{ Hamiltonian of } S).$$

The *Principle of Diminishing Potentialities (PDP)* is the statement that

$$\boxed{\mathcal{E}_{\geq t} \underset{\neq}{\supset} \mathcal{E}_{\geq t'}\,, \;\; \text{whenever } \; t' > t \geq t_0.} \tag{10}$$

This principle characterizes *isolated open* systems, namely systems that can release *"events"* to the outside world. It has been proven to hold in axiomatic QFT ($\nearrow$ *Buchholz*) and in simple models of measurements ...

The analogue of the initial position x_0 of a Brownian test particle at time t_0 is a projection $\pi_0 \in \mathcal{E}_{\geq t_0}$ giving rise to a state $\Omega_{t_0} = \frac{\pi_0}{\mathrm{tr}[\pi_0]}$. A **state**, Ω_t, at time $t > t_0$ is a *quantum probability measure* on the *lattice of all potential events in* $\mathcal{E}_{\geq t}$, which (by theorems of *Gleason* and *Maeda*) is the same as a normal state on $\mathcal{E}_{\geq t}$. Note: Ω_t not nec. a density matrix!

Actual events

Given a state Ω_t at time t, its *centralizer*, $\mathcal{C}_{\Omega_t}$, is the subalgebra of $\mathcal{E}_{\geq t}$ generated by all potential events $\{\pi_\xi \,|\, \xi \in \mathfrak{X}\} \subset \mathcal{E}_{\geq t}$ commuting with Ω_t; i.e., $\Omega_t = \sum_{\xi \in \mathfrak{X}} \pi_\xi \Omega_t \pi_\xi$.

Let $\mathcal{Z}_{\Omega_t}$ be the abelian subalgebra of $\mathcal{C}_{\Omega_t}$ generated by all potential events $\{\pi_\xi \,|\, \xi \in \mathfrak{X}\} \in \mathcal{C}_{\Omega_t}$ commuting with *all* the operators in the centralizer $\mathcal{C}_{\Omega_t}$. It is easy to see that $\mathcal{Z}_{\Omega_t}$ is generated by the projections of *one* potential event, $\{\pi_\xi \,|\, \xi \in \mathfrak{X}_{\Omega_t}\} \in \mathcal{C}_{\Omega_t}$.

Definition 2 (Actualities): $\mathfrak{e}_{\Omega_t} := \{\pi_\xi \,|\, \xi \in \mathfrak{X}_{\Omega_t}\}$ is the potential event **actualizing** at time t, given that the state of S at time t is Ω_t.

We are now prepared to introduce a **Law** governing the **stochastic time evolution** of the state of an individual isolated system S. In order to be precise, mathematically, we temporarily suppose that *time is discrete*, i.e., $t \in \mathbb{Z}_\tau$, where $\tau > 0$ is an elementary time step. (If the algebras $\mathcal{E}_{\geq t}, t \in \mathbb{R}$, are type-I one can let $t \to 0$. ...)
Thanks to *PDP* and the phenomenon of **entanglement**, the following *State-Reduction Postulate* is meaningful and non-trivial:
Suppose that Ω_t is the state of S at time t; we define $\overline{\Omega}_{t+\tau}$ to be the state on $\mathcal{E}_{\geq t+\tau}$ obtained by *restriction* of Ω_t to $\mathcal{E}_{\geq t+\tau} \underset{PDP}{\subsetneq} \mathcal{E}_{\geq t}$.

The state-reduction postulate

Axiom CP: *Let* $\mathfrak{e}_{\overline{\Omega}_{t+\tau}} = \{\pi_\xi \,|\, \xi \in \mathfrak{X}_{\overline{\Omega}_{t+\tau}}\}$ *be the potential event actualizing at time* $t+\tau$*, given the state* $\overline{\Omega}_{t+\tau}$*.*

Then **'Nature'** *replaces the state* $\overline{\Omega}_{t+\tau}$ *on* $\mathcal{E}_{\geq t+\tau}$ *by a state*

$$\Omega_{t+\tau} \equiv \Omega_{t+\tau,\xi} := \left[\mathrm{tr}(\overline{\Omega}_{t+\tau}\,\pi_\xi)\right]^{-1} \cdot \pi_\xi \overline{\Omega}_{t+\tau} \pi_\xi \,, \qquad (12)$$

for some $\pi_\xi \in \mathfrak{e}_{\overline{\Omega}_{t+\tau}}$*, with* $\mathrm{tr}\big(\overline{\Omega}_{t+\tau}\,\pi_\xi\big) \neq 0$.

The probability, $prob_{t+\tau}(\xi)$*, for the state* $\Omega_{t+\tau,\xi}$ *in (12) to be selected by* **'Nature'** *as the state of* S *at time* $t+\tau$ *is given by*

$$prob_{t+\tau}(\xi) = \mathrm{tr}[\overline{\Omega}_{t+\tau}\,\pi_\xi] \qquad \textit{(generalized Born Rule)} \qquad \square \qquad (13)$$

The projection $\pi(t+\tau) := \pi_\xi \in \mathcal{Z}_{\overline{\Omega}_{t+\tau}}$ appearing in (12) is called **actual event**, or *"actuality"*, at time $t+\tau$.
With a **history**, $\{\pi(t_0+\tau), \ldots, \pi(t)\}$ of actualities (given an initial event π_0 at time t_0) we associate a *"history operator"*

$$H_{\pi_0}(t_0, t) := \pi_0 \prod_{t_0 < t' \leq t} \pi(t'), \qquad \text{with} \quad t' \in \mathbb{Z}_\tau \,.$$

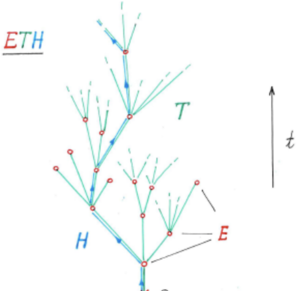
Predicting the probabilities of histories

The analogue of the Wiener measure, dW_{x_0}, is given by

$$prob_{\pi_0}\{\pi(t_0+\tau),\ldots,\pi(t)\} := \frac{\mathrm{tr}\left[H_{\pi_0}(t_0,t)^* \cdot H_{\pi_0}(t_0,t)\right]}{\mathrm{tr}[\pi_0]} \qquad (14)$$

Apparently, the time-evolution of the *state* of a physical system S is described by a *stochastic branching process* (a *"quantum Poisson process"*), with branching rules as determined by **Axiom CP**. This is meaningful, mathematically, if $\tau > 0$; but, for the time being, the limiting theory, as $\tau \searrow 0$, is only well understood in the type-I case; (e.g., model of spontaneous emission of photons in the limit $c \to \infty$).

A metaphorical picture:



3. Huygens' Principle and *PDP*

"... principles are tested by inferences which are derivable from them."

(Christiaan Huygens)

Fact: *Huygens' Principle* for massless modes (photons, gravitons, ...) in isolated physical systems

$\Rightarrow$ *Principle of Diminishing Potentialities !*

Example: S an isolated system consisting of a *static atom* located near $\mathbf{x} = 0$, coupled to the *electromagnetic field*.

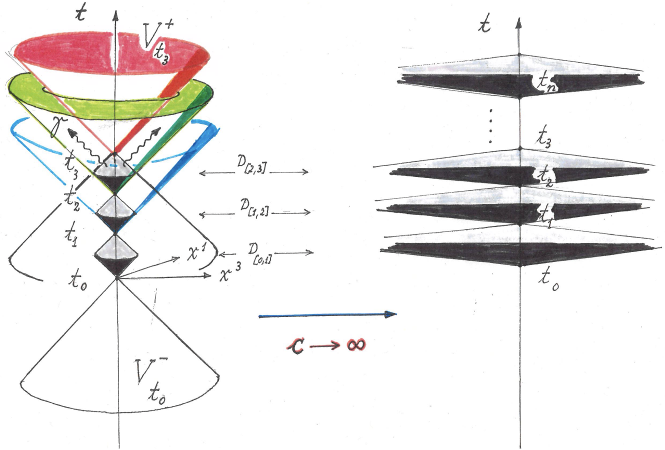
- Atom has M energy levels, Hilbert space $\mathfrak{h}_A \simeq \mathbb{C}^M$.
- Hilbert space of free e.m. field = *Fock space*, $\mathfrak{F}$, of photons; the e.m. field is described by field tensor, $F_{\mu\nu}(\tau, \mathbf{x})$, with the property that, for real-valued test fus. $\{h^{\mu\nu}\}$ on space-time,

$$F(h) := \int_{\mathbb{R}\times\mathbb{R}^3} d\tau \, d\mathbf{x} \, F_{\mu\nu}(\tau, \mathbf{x}) \, h^{\mu\nu}(\tau, \mathbf{x})$$

is a self-adjoint op. on $\mathfrak{F}$ and satisfies **locality**. The usual Hamiltonian of the free e.m. field is denoted by H_f; with $H_f = H_f^* \geq 0$ on $\mathfrak{F}$.

Light cones, space-time diamonds, time slices

We consider space-time diamonds $D_{[t,t']} := V_t^+ \cap V_{t'}^-$, $t' > t$, centered on time axis ($\mathbf{x} = 0$), and time slices arising as $c \to \infty$:



space-time diamonds c= velocity of light time slices in NR limit, $c \to \infty$

Next time: $c \to \infty$.

A concrete model of S

Hilbert space of S:

$$\mathcal{H} := \mathfrak{F} \otimes \mathfrak{h}_A .$$

Bounded functions of field operators $F(h)$, $\text{supp}(h^{\mu\nu}) \subseteq D_{[t,t']}$, generate a von Neumann algebra $\mathcal{A}_{I=[t,t']}$. We define algebras

$$\mathcal{D}_I^{(0)} := \mathcal{A}_I \otimes \mathbf{1}\big|_{\mathfrak{h}_A}, \qquad \mathcal{E}_I^{(0)} := \mathcal{A}_I \otimes B(\mathfrak{h}_A),$$
$$\mathcal{E}_{\geq t}^{(0)} := \overline{\bigvee_{I \subset [t,\infty)} \mathcal{E}_I^{(0)}} . \tag{15}$$

PDP holds for the non-interacting system: Setting $I := [t, t']$, one has

$$\boxed{\left[\mathcal{E}_{\geq t'}^{(0)}\right]' \cap \mathcal{E}_{\geq t}^{(0)} = \mathcal{D}_I^{(0)} \quad (\text{an } \infty - \text{dim. algebra}) .} \tag{16}$$

Remark: This follows from *"Huygens' Principle"*, namely from

$$[F_{\mu\nu}(x), F_{\rho,\sigma}(y)] = 0, \ \text{ unless } x - y \text{ is } \textbf{light-like},$$

(see figure). From now on, we make use of an ultraviolet regularization of QED arising from discretizing time: $t_n := n\tau, n \in \mathbb{Z}, \tau > 0$.

PDP for an interacting model

To describe interactions, we pick a unitary op. $U \in \mathcal{E}^{(0)}_{[0,1]}$ and define

$$U_k := e^{i(k-1)\tau H_f}\, U\, e^{-i(k-1)\tau H_f}, \quad k = 1, 2, \ldots, \quad U(n) := \prod_{k=1}^{n} U_k\,,$$

$$\Gamma := e^{-i\tau H_f}\, U \;\Rightarrow\; \Gamma^n = e^{-in\tau H_f}\, U(n),\; (\Gamma^n)^* = \Gamma^{-n},\; n = 0, 1, 2, \quad (17)$$

$\{\Gamma^n\}_{n\in\mathbb{Z}}$: propagator of interacting systems with discrete time.

It suffices to consider time evolution for times $t \geq t_0 := 0$. Define

$$\mathcal{E} := \mathcal{E}^{(0)}_{\geq 0}, \quad \mathcal{E}_{\geq n} := \left\{\Gamma^{-n}\, X\, \Gamma^n \,\middle|\, X \in \mathcal{E}\right\}. \quad (18)$$

Verification of PDP for interacting model: Using (17) and (18), one readily shows that

$$\left[\mathcal{E}_{\geq n'}\right]' \cap \mathcal{E}_{\geq n} \simeq \mathcal{D}_{[n,n']}\,, \;\text{ for }\; n' > n, \quad (19)$$

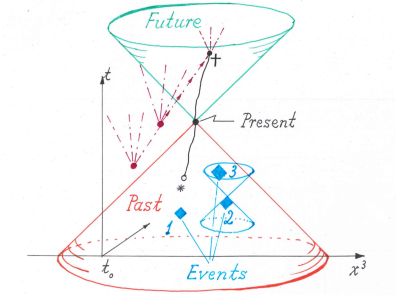
where $\mathcal{D}_{[n,n']} := \left\{U(n')^*\, X\, U(n') \,\middle|\, X \in \mathcal{D}^{(0)}_{[n,n']}\right\}$.

Preparing the system in an initial state Ω_0 at time $n = 0$, one determines the time evolution of its state according to the *ETH*- Approach, as prescribed in *Definition 2* – actualities – and **Axiom CP** of Sect. 2. Explicit results are hard to derive because of memory effects. ...

4. A sketch of relativistic quantum theory

To begin with, I recall that, usually, relativistic theories, whether classical or quantum, do not fully predict the future.

Example: Space-time with an *event horizon:* For fundamental reasons, an *observer* sitting at *Present* does not have complete access to the initial condition, hence does **not** know of dangers lurking from outside his *past light-cone* – might get killed at †.



t_0: time right after inflation → event horizon ⇒ initial conditions not fully accessible!

Past = History of Events (Facts) / Future = Ensemble of Potentialities

This Aristotelian dichotomy is realized perfectly in Quantum Mechanics!

The Past - Future Dichotomy in relativistic *QM*

For the time being, space-time is given by $\mathbb{M}^4$; (with gravity turned off).

For $P \in \mathbb{M}^4$, V_P^+ denotes the forward (future) light cone and V_P^- the backward (past) light cone with apex in P.

A "diamond," D, is defined by $D = V_P^+ \cap V_{P'}^-$, with $P' \in V_P^+$.
With every diamond $D \subset \mathbb{M}^4$ we associate a von Neumann algebra $\mathfrak{A}(D)$ of *"potentialities"* rep. by operators localized in D. The net $\left\{\mathfrak{A}(D) \,\middle|\, D \subset \mathbb{M}^4\right\}$ is assumed to have all the usual properties assumed in algebraic QFT, including *Landau's property*:

$$D_1 \cap D_2 = \emptyset \quad \Rightarrow \quad \mathfrak{A}(D_1) \cap \mathfrak{A}(D_2) = \emptyset\,. \tag{15}$$

Given a forward light cone $V^+ \subset \mathbb{M}^4$, we define

$$\mathfrak{A}(V^+) := \bigvee_{D \subset V^+} \mathfrak{A}(D)\,.$$

($\mathfrak{A}(V^+)$ is **not** taken to be norm-closed!) Landau's property shows that

$$\text{if} \quad D \cap V^+ = \emptyset \quad \text{then} \quad \mathfrak{A}(D) \cap \mathfrak{A}(V^+) = \emptyset\,. \tag{16}$$

For theories with massless modes, (16) may hold even if $\mathfrak{A}(V^+)$ were weakly closed. (I refer you to the lecture by *Detlev Buchholz*.)

Potential events and their supports

Potential events, $\mathfrak{e}$, are defined to be partitions of unity by disjoint orthogonal projections, π_ξ,

$$\mathfrak{e} = \{\pi_\xi \,|\, \xi \in \mathfrak{X}_{\mathfrak{e}}\}, \quad \pi_\xi \cdot \pi_\eta = \delta_{\xi\eta}\pi_\xi,$$

belonging to an algebra $\mathfrak{A}(D)$, for some diamond D.

We define the *"support,"* $D_{\mathfrak{e}}$, of a potential event $\mathfrak{e}$ to be the smallest diamond with the property that $\mathfrak{e} \subset \mathfrak{A}(D_{\mathfrak{e}})$. Given a potential event $\mathfrak{e}$ with support $D_{\mathfrak{e}}$, we let $V^+_{\mathfrak{e}}$ to be the smallest forward light cone containing $D_{\mathfrak{e}}$. We use the notations

$$\mathfrak{A}_{\mathfrak{e}} := \mathfrak{A}(D_{\mathfrak{e}}), \qquad \mathfrak{A}^+_{\mathfrak{e}} := \mathfrak{A}(V^+_{\mathfrak{e}})\,. \tag{17}$$

Definition:

1. $\mathfrak{e}$ is in the **past** of $\mathfrak{e}'$, written as $\mathfrak{e} \prec \mathfrak{e}'$, iff

$$\mathfrak{A}_{\mathfrak{e}'} \subset \mathfrak{A}^+_{\mathfrak{e}} \tag{18}$$

If $\mathfrak{e} \prec \mathfrak{e}'$ we say that $\mathfrak{e}'$ is in the **future** of $\mathfrak{e}$, written as $\mathfrak{e}' \succ \mathfrak{e}$.

Actual events

2. If $\mathfrak{e}_1 \nprec \mathfrak{e}_2$ and $\mathfrak{e}_2 \nprec \mathfrak{e}_1$ and if all operators in $\mathfrak{A}_{\mathfrak{e}_1}$ commute with all operators in $\mathfrak{A}_{\mathfrak{e}_2}$ we say that $\mathfrak{e}_1$ and $\mathfrak{e}_2$ are **space-like separated**, written as $\mathfrak{e}_1 \text{ X } \mathfrak{e}_2$.
3. Let Σ be a space-like surface contained in a backward light cone V^-. An **initial state**, ω_Σ, in the future of Σ is a state on the algebra

$$\mathfrak{A}^+_\Sigma := \bigvee_{P \in \Sigma} \mathfrak{A}(V^+_P)\,. \tag{19}$$

I will temporarily assume that the conditions for a potential event to *actualize* are known; (see below; but the details are too lengthy to be presented here.) Let $\mathfrak{e}_0$ be a potential event in $\mathfrak{A}^+_\Sigma$, and let $\mathfrak{F}_{\mathfrak{e}_0} \subset \mathfrak{A}^+_\Sigma$ be the family of all events in the **past** of $\mathfrak{e}_0$ that have actualized. For every $\mathfrak{e} \in \mathfrak{F}_{\mathfrak{e}_0}$, $\exists$ an orthogonal projection $\pi \in \mathfrak{e}$ representing an **actual event** (or "fact"). We impose the following fundamental

Postulate 1: For every pair $(\mathfrak{e}, \mathfrak{e}') \subset \mathfrak{F}_{\mathfrak{e}_0}$,

$$\text{either } \mathfrak{e} \prec \mathfrak{e}', \quad \text{or } \mathfrak{e}' \prec \mathfrak{e}, \quad \text{or } \mathfrak{e} \text{ X } \mathfrak{e}'. \tag{20}$$

History operators

By (20) the following **history operators** are well defined:

$$H_{\mathfrak{F}_{\mathfrak{e}_0}}(\underline{\pi}) := \overrightarrow{\prod_{\mathfrak{F}_{\mathfrak{e}_0}\ni\mathfrak{e}\ni\pi}} \pi \,, \quad \text{with} \quad \underline{\pi} := \bigcup_{\mathfrak{F}_{\mathfrak{e}_0}\ni\mathfrak{e}\ni\pi} \pi \,, \tag{21}$$

where $\overrightarrow{\prod}$ means that $\pi \in \mathfrak{e}$ is to the right of $\pi' \in \mathfrak{e}'$ if $\mathfrak{e} \prec \mathfrak{e}'$; and, if $\mathfrak{e} \text{ X } \mathfrak{e}'$ then the ordering of π and π' is irrelevant.

A state $\omega_{\mathfrak{e}_0}$ on the algebra $\mathfrak{A}^+_{\mathfrak{e}_0}$ is defined by

$$\omega_{\mathfrak{e}_0}(X) := \frac{\omega_\Sigma\left(H_{\mathfrak{F}_{\mathfrak{e}_0}}(\underline{\pi})^* \cdot X \cdot H_{\mathfrak{F}_{\mathfrak{e}_0}}(\underline{\pi})\right)}{\omega_\Sigma\left(H_{\mathfrak{F}_{\mathfrak{e}_0}}(\underline{\pi})^* \cdot H_{\mathfrak{F}_{\mathfrak{e}_0}}(\underline{\pi})\right)}, \qquad \forall X \in \mathfrak{A}^+_{\mathfrak{e}_0} \,. \tag{22}$$

A necessary (but, in view of (20) not sufficient) condition for $\mathfrak{e}_0$ to actualize is the following

Postulate 2: A necessary condition for the potential event $\mathfrak{e}_0$ to actualize is that $\mathfrak{e}_0$ belongs to the *center* of the *centralizer* of the state $\omega_{\mathfrak{e}_0}$ on the algebra $\mathfrak{A}^+_{\mathfrak{e}_0}$.

The feasibility of this condition derives from a basic result due to *Buchholz:* If there are massless particles in the theory then, for P' in the future of P, $\mathfrak{A}(V^+_{P'})' \cap \mathfrak{A}(V^+_P)$ is an infinite-dimensional algebra.

The relativistic Born Rule

We are now able to assign a **probabilty** to a given history

$$h(\pi_0, \underline{\pi}) := \left\{\pi_0 \in \mathfrak{e}_0, \underline{\pi} \subset \mathfrak{F}_{\mathfrak{e}_0}\right\}.$$

The probability of a history $h(\pi_0, \underline{\pi})$, assuming that the initial state is ω_Σ and that one sums over *all possible events* that are space-like separated from $\mathfrak{e}_0$, is given by

$$\underbrace{prob\big[h(\pi_0, \underline{\pi})\big] = \omega_\Sigma\left(H_{\mathfrak{F}_{\mathfrak{e}_0}}(\underline{\pi})^* \, \pi_0 \, H_{\mathfrak{F}_{\mathfrak{e}_0}}(\underline{\pi})\right)}_{\text{Relativistic } \textit{Born Rule}} \tag{23}$$

Obviously $prob\big[h(\pi_0, \underline{\pi})\big]$ is non-negative, and

$$\sum_{\pi_0 \in \mathfrak{e}_0, \underline{\pi} \subset \mathfrak{F}_{\mathfrak{e}_0}} prob\big[h(\pi_0, \underline{\pi})\big) = 1 \,.$$

Remarks:

(i) For an arbitrary $X \in \mathfrak{A}^+_{\mathfrak{e}_0}$ and an arbitrary $\pi_0 \in \mathfrak{e}_0$,

$$\sum_{\underline{\pi} \subset \mathfrak{F}_{\mathfrak{e}_0}} \omega_\Sigma\left(H_{\mathfrak{F}_{\mathfrak{e}_0}}(\underline{\pi})^* \, \pi_0 \, X \, \pi_0 \, H_{\mathfrak{F}_{\mathfrak{e}_0}}(\underline{\pi})\right) = \omega_\Sigma\left(\pi_0 \, X \, \pi_0\right). \tag{24}$$

Ontology, and meaning of states

(ii) The prediction of the probability of a history will be modified as one moves into the future of $\mathfrak{e}_0$, because then events space-like separated from $\mathfrak{e}_0$ will appear in backward light cones and will have an effect on the probability predicted for $h(\pi_0, \underline{\pi})$. However, summing over all possible events space-like separated from $\mathfrak{e}_0$ will restore the prediction (23)!

(iii) The **ontology** of relativistic quantum theory lies in histories of events satisfying Postulates 1 and 2.

(iv) A state $\omega_{\mathfrak{e}_0}$ enables one to predict the expected probabilty of potential events in the future of $\mathfrak{e}_0$ to actualize when an average over identical systems prepared in the state $\omega_{\mathfrak{e}_0}$ is taken. The state is merely a mathematical device to calculate probabilities.

(v) Postulate 1 imposes a very strong, seemingly **non-local** constraint on actual events (i.e., on the actualization of pot. events). The significance of this postulate should be understood more clearly. In contrast, Postulate 2 is very natural and enables one to show that averages of expectation values of future pot. events over past histories are given by the usual expressions known from text books.

To conclude some remarks

1. It appears possible to eliminate any reference to a specific model of space-time and reformulate everything in Sect. 3 in terms of relations between algebras $\mathfrak{A}_{\mathfrak{e}}$ and $\mathfrak{A}_{\mathfrak{e}}^{+}$ for different potential events $\mathfrak{e}$.
2. Postulate 1 and 2 of Sect. 3 **cannot** be expected to uniquely determine the events actualizing inside any past light cone. In view of Remark 1, I expect this ambiguity to express the effects of

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   Details to be understood more precisely.
3. The usual Schrödinger-Liouville evolution of states arises when one takes an average over all possible histories.
4. We have applied the non-relativistic *ETH* - approach, as developed in Sect. 2, to various concrete systems, such as:
   - Models of *measurements of physical quantities* characteristic of a subsystem of an isolated system; (works perfectly!).
   - Spontaneous emission of photons from atoms prepared in excited states; i.e, *fluorescence* of atoms $\rightarrow$ predictions that can in principle be confirmed or refuted in experiments!

I migliori auguri a tutte e tutti di pace e benessere!

5. In work with *B. Schubnel*, I have proposed concrete models describing the preparation of specific initial states of isolated systems.

Picasso

Thanks for listening!