The Quantum Mechanics of Particle Tracks in Detectors[1]

A semi-classical analysis of the quantum dynamics of particles moving through detectors

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Summary of this talk

I consider regimes of Quantum Mechanics, which I call *"Classical Periphery of QM"* (CPQM), that can be described in essentially classical terms.

I illustrate some general ideas of CPQM in a study of *tracks traced out by quantum-mechanical particles* propagating in detectors. These tracks are close to classical particle trajectories.

I will begin my talk with some general comments on the notion of *"events"* in Quantum Mechanics and their role in understanding *"state reduction"* (as manifested, e.g., in measurements). My discussion is cast in what I have dubbed *"ETH-Approach to QM"*, an attempt to complete QM to a consistent theory.

C.G. Darwin

N. Mott

Credits and Contents

- *Thanks are due to:* M. Ballesteros, T. Benoist, Ph. Blanchard, N. Crawford, M. Fraas, A. Pizzo, and B. Schubnel for collaborations; *and to:* M. Bauer, F. Finster, S. Goldstein, B. Kümmerer, C. Paganini, and others – for useful discussions.

  I wish to draw atention to much interesting work on particle tracks by R. Figari and A. Teta.

- Here is a table of *Contents* of this talk:

"The interpretation of quantum mechanics (QM) has been dealt with by many authors, and I do not want to discuss it here. I want to deal with more fundamental things" – P.A.M. Dirac

1. Comments on the Foundations of Quantum Mechanics

"... their attempts to see in the very inadequacy of the conventional interpretation of quantum theory a deep physical principle have often led physicists to adopt obscurantist, mystical, positivist, psychical, and other irrational worldviews." (David Deutsch)

This talk touches upon the *foundations of Quantum Mechanics*, a subject that, in my opinion, *ought to occupy center stage of contemporary theoretical physics*.
Unfortunately, many people who think about quantum foundations prefer to endlessly talk about puzzles and paradoxes and the "weirdness" of Quantum Mechanics – rather than to try to actually solve some of the most pressing open problems, such as the so-called **"Measurement Problem"**, the **"Information Paradox"**, or a coherent account of **Relativistic Quantum Theory** – problems that, I am convinced, can actually be addressed successfully.

Thus, let's get started!

Fundamental questions about QM

"It seems clear that the present quantum mechanics is not in its final form." (P.A.M. Dirac)

In our courses, we tend to describe the QM of a physical system, S, in terms of a *Hilbert space*, $\mathcal{H}_S$, of pure state vectors and a *unitary propagator*, $(U_S(t,s))_{t,s\in\mathbb{R}}$, describing the "time evolution" of its states.

Unfortunately, these data hardly encode *any* interesting information about S that would enable one to draw conclusions about its physical properties, and they give the **erroneous** impression that QM might be a *deterministic theory* with *linear time evolution*.

$\longrightarrow$ Fundamental questions and problems:

1. What do we have to **add** to the usual formalism of QM to arrive at a mathematical structure that, through interpretation (?), can be given unambiguous physical meaning – *without the intervention* of "observers"?

2. Where does the *fundamental randomness* observed in the behavior of microscopic systems come from, given the deterministic character of the Schrödinger- and Heisenberg equations? How does it differ from classical randomness?

More questions and some claims

In trying to answer these questions one meets further questions:

3. What are *potential events* & *actual events* in QM? What is the relation between *actual events* and **state reduction** (wave-function collapse)?
4. What is the *time evolution* of *physical quantities* and of *states* of individual systems in the **Heisenberg picture**?

Some basic definitions and claims:

i. *Physical quantities* ("observables") of a system S are repr. by *linear operators* acting on a separable Hilbert space.

ii. An *isolated system*, S, is one that has negligible interactions with its complement, i.e., with the rest of the Universe.
In the QM of S, the *Heisenberg (-picture) time evolution* of "observables" makes perfect sense and does not depend on knowledge of the complement of S.

More claims

Yet, nothing could be farther from the truth than the claim that the Schrödinger equation yields a correct description of the time evolution of states of an individual isolated system featuring events!

iii. In non-relativistic QM, *potential events* of S are described by partitions of unity by disjoint orthogonal projections. All potential events possibly setting in at some time t or later generate algebra, $\mathcal{E}_{\geq t}$.[2] An isolated system is characterized by a "co-filtration," $\left\{\mathcal{E}_{\geq t}\right\}_{t\in\mathbb{R}}$, of such algebras. A *state* of S at time t is defined to be a **quantum probability measure** (= normal state) on $\mathcal{E}_{\geq t}$.

An *event actualizing* at time t is a potential event in $\mathcal{E}_{\geq t}$ whose adjoint action on the state of S at time t vanishes ...

iv. An *isolated open system* S, i.e., one releasing *actual events*, has the property that

$$\boxed{\mathcal{E}_{\geq t} \underset{\neq}{\supset} \mathcal{E}_{\geq t'}, \quad \text{whenever} \quad t' > t.}$$

This expresses the *Principle of Diminishing Potentialities (PDP)*.

[2] a von Neumann algebra, to be precise

The *ETH* - Approach to QM

(PDP) can be shown to hold in certain *relativistic quantum theories* with massless modes, such as photons and/or gravitons. But it is expected that there are plenty of other mechanisms implying *(PDP)*.

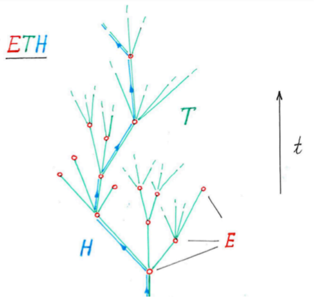
v. When combined with *entanglement*, *(PDP)* yields a *stochastic law* for the *time evolution* of *states* of *individual* systems that involves **state reduction** (w.f. collapse) governed by *Born's Rule*. This replaces *Schrödinger evolution*. It yields a *probabilistic completion of QM* apt to describe **histories** of *actual events* released by individual systems.

vi. A **projective measurement** amounts to retrieving info about *S* by recording sequences of *actual events* released by it – *actual events are a corner stone in a theory of measurements!*

The details underlying these claims form the basis of the so-called *"ETH - Approach to QM"* (for *"Events, Trees, and Histories"*).

The evolution of states in the *ETH* - Approach

Here is a metaphoric picture of the evolution of states according to the *ETH* - Approach:



E: "Events", *T*: "Trees" of possible states, *H*: "Histories" of states

Upshot: In QM, the evolution of *states* of *individual* systems featuring actual events is described in terms of a new kind of *stochastic branching process* – *replacing Schrödinger evolution* – whose non-commutative state space is defined in terms of partitions of unity by disjoint orthogonal projections in a universal von Neumann algebra; the branching rules are determined by *Born's Rule*. – *Ontology = History of Actual Events!*

2. Indirect Measurements in Quantum Mechanics

In QM, information about a physical system, *S*, of interest is gained by measurements describable in the *classical periphery* of QM. Often, information on properties of *S* is gathered by **indirect measurements** involving **probes** (photons, neutrons, atoms, etc.) that interact with *S*, their states getting entangled with the state of *S*. After their interaction with *S* the probes are subjected to **projective measurements** describable within the *ETH* - Approach to QM.

Plato's allegory of the cave as an illustration of indirect acquisition of information in QM

The example of Mott tracks

Because of entanglement, a long sequence of (possibly very boring) projective measurements of probes yields (possibly very interesting) information on the *state* of S.

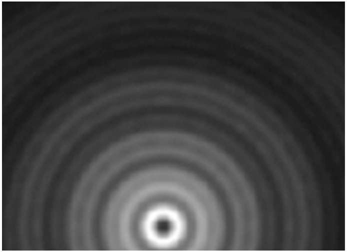
Given a theory of *projective measurements*, as provided by the *ETH* - Approach to QM, the general theory of *indirect measurements*[3] is well developed. I propose to illustrate it by explaining how, classical-looking tracks of particles interacting with the degrees of freedom of a detector (that are then subjected to projective measurements!) can be understood to appear.

History: At the 1927 Solvay conference, in a famous debate with Bohr and Born, the problem of the classical periphery of QM, and in particular the problem of particle tracks, was raised by Einstein. Born sketched an insightful answer due to Heisenberg. – Later it was studied by C. G. Darwin and N. Mott, whence the name *"Mott tracks"*.

More recent work is due to Blasi et al.; O. Steinmann; R. Figari and A.Teta (who have included a great account of the history); BBFF, BFF ...

[3] Kraus, Maassen and Kümmerer, and others

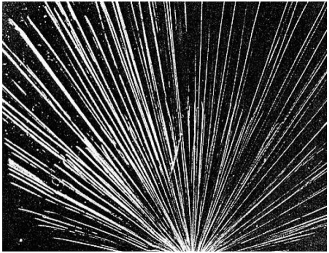
Mott tracks of α-particles emitted by radioactive ball



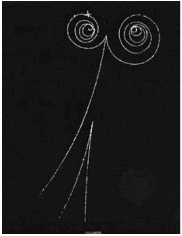
α-waves in a dark cavity

$\Rightarrow$

Effect of interactions in detector



α-particle tracks in a bubble chamber



Electrons and positrons produced simultaneously from individual gamma rays curl in opposite directions in the magnetic field of a bubble chamber. In the above example the gamma ray has lost some energy to an atomic electron, which leaves the long track, curling left. The gamma rays do not leave tracks in the chamber, as they have no electric charge.

3. Particle Tracks in Detectors – BFF

"Every experiment destroys some of the knowledge of the system which was obtained by previous experiments." (Werner Heisenberg)

3.1. QM of a charged particle, semi-classical regime

Consider *quantum dynamics* of massive charged particle prepared in an initial state of very high kinetic energy, moving in an ext. magnetic field & periodically illuminated by laser pulses. Scattered light is assumed to hit detectors that click with high probability when a photon arrives. Our goal is to show that, no matter what exactly the initial state of the particle is, the observed approximate particle positions are close to points on the trajectory of a **classical** charged particle of the same mass and charge moving in the same magnetic field.

Hilbert space and Hamiltonian of particle:

$$\mathcal{H} := L^2(\mathbb{R}^3, d^3x) \tag{1}$$

$$H := \frac{1}{2M}\big[P - eA(X)\big]^2 + V(X), \tag{2}$$

where $M = \mathcal{O}(1)$ is the mass and e the electric charge of the particle, $A =$ vector potential of a time-indep., c-number ext. magnetic field B,

Commutation relations, and semi-classical regime

$V =$ ext. potential; $P =$ momentum operator, $X =$ position operator, which satisfy the Heisenberg commutation relations (CCR)

$$\left[X_i, P_j\right] = i\hbar\delta_{ij}\mathbf{1}, \qquad \left[X_i, X_j\right] = \left[P_i, P_j\right] = 0, \quad i, j = 1, 2, 3. \tag{3}$$

$\Psi_t =$ state of particle at time t, assumed to satisfy the Schrödinger eq.

$$i\hbar\frac{\partial}{\partial t}\Psi_t = H\Psi_t \tag{4}$$

when there is no light scattering (lasers turned off).

Semi-classical regime: Kinetic energy in initial state, Ψ_0, is very large; i.e.,

$$\frac{1}{2M}\langle\Psi_0, \left(P - eA\right)^2\Psi_0\rangle = \mathcal{O}(\varepsilon^{-1}\hbar\,\omega) \tag{5}$$

$\omega =$ frequency of laser light. Hence average of *speed* of particle in Ψ_0 is $\mathcal{O}(\varepsilon^{-1/2})$, with $0 < \varepsilon < \varepsilon_0 \ll 1$. $\rightarrow$ Re-scale momentum- and position operators:

$$P =: \varepsilon^{-1/2}\widehat{p}, \quad X =: \varepsilon^{-1/2}\widehat{x}, \quad \text{with} \quad \left[\widehat{x}_i, \widehat{p}_j\right] = i\varepsilon\hbar\,\delta_{ij}\mathbf{1}, \tag{6}$$

other commutators $= 0$. From now on $\hbar = 1$. *Classical limit:* $\varepsilon \searrow 0$.

Semi-classical regime, ctd.

Choose $A \equiv A_\varepsilon$, $V \equiv V_\varepsilon$ to depend on ε in such a way that

$$A_\varepsilon(\varepsilon^{-1/2}\widehat{x}) \sim \varepsilon^{-1/2} A_0(\widehat{x}), \quad V_\varepsilon(\varepsilon^{-1/2}\widehat{x}) \sim \varepsilon^{-1} V_0(\widehat{x}), \quad \text{as} \;\; \varepsilon \searrow 0. \qquad (7)$$

In 3D, this choice of A_ε automatically holds for the vector pot. of a uniform magnetic field, $B \in \mathbb{R}^3$, i.e., for $A(X) = \frac{1}{2}(X \wedge B)$; and the formula for V_ε automatically holds for a harmonic potential, $V(X) \propto |X|^2$.

If the relations in (7) hold, the Schrödinger eq. reads

$$i\varepsilon \frac{\partial}{\partial t}\Psi_t = \widehat{H}\Psi_t\,, \;\; \text{with} \quad \widehat{H} := \Big[\frac{1}{2M}\big(\widehat{p} - eA_0(\widehat{x})\big)^2 + gV_0(\widehat{x})\Big]. \qquad (8)$$

The semi-classical regime corresponds to $\varepsilon \ll 1$ and initial states, $\Psi_0 \in \mathcal{H}$, with the properties that $\|\Psi_0\|_2 = 1$, and

$$\Delta_{\Psi_0}\hat{x} \cdot \Delta_{\Psi_0}\hat{p} = \mathcal{O}(\varepsilon),$$

where

$$\Delta_{\Psi_0} A := \sqrt{\langle \Psi_0, \big(A - \langle A \rangle_{\Psi_0}\big)^2 \Psi_0 \rangle} \quad \text{and} \quad \langle A \rangle_{\Psi_0} := \langle \Psi_0, A\Psi_0 \rangle.$$

3.2. How QM arises from CM by Weyl quantization

Phase space of classical particle is: $\Gamma := \mathbb{R}^d_x \oplus \mathbb{R}^d_p$ (with $d = 3$), points in Γ are denoted by $\xi, \zeta, \ldots$. Furthermore, $\mathcal{C}(\Gamma)$ = space of bounded, smooth functions on Γ.
The Fourier transform, $\mathcal{F}(a)$, of a function $a \in \mathcal{C}(\Gamma)$ is given by

$$\mathcal{F}(a)(\zeta) \equiv \widetilde{a}(\zeta) := (2\pi)^{-d} \int_\Gamma a(\xi)\, e^{-i\xi\cdot\Omega\,\zeta} d\xi\,, \quad \text{with} \tag{9}$$

$$\xi := (x, p), \quad \zeta := \begin{pmatrix} \mathfrak{x} \\ \mathfrak{p} \end{pmatrix}, \quad \Omega = \begin{pmatrix} 0 & -\mathbf{1}_d \\ \mathbf{1}_d & 0 \end{pmatrix}.$$

Weyl quantization: $a \mapsto \widehat{a} \equiv \mathrm{Op}_\varepsilon(a)$, of $a \in \mathcal{C}(\Gamma)$ is defined by

$$\widehat{a} \equiv \mathrm{Op}_\varepsilon(a) := (2\pi)^{-d} \int_\Gamma \widetilde{a}(\zeta)\, W(\zeta)\, d\zeta, \qquad \text{where} \tag{10}$$

$$W(\zeta) \equiv W_\varepsilon(\zeta) := \exp\left[i(\widehat{\xi} \cdot \Omega\zeta)\right], \qquad \zeta := \begin{pmatrix} \mathfrak{x} \\ \mathfrak{p} \end{pmatrix} \text{ and } \widehat{\xi} := (\widehat{x}, \widehat{p})\,, \tag{11}$$

are the *Weyl operators*, and $\hat{x}$, $\hat{p}$ are the position- and momentum ops., resp., on $\mathcal{H}$, satisfying (6).

Weyl quantization - ctd.

The Weyl operators $W(\zeta), \zeta \in \Gamma$, are unitary & satisfy *Weyl relations*

$$W(\zeta_1)\, W(\zeta_2) = e^{i\frac{\varepsilon}{2}\,(\zeta_1^t \cdot \Omega\, \zeta_2)}\, W(\zeta_1 + \zeta_2) \tag{12}$$

Note that

$$W(\zeta)^* = W(-\zeta) \quad \text{and} \quad W(0) = \mathbf{1} \quad \Rightarrow \quad \widehat{a}^* = \widehat{a}\,, \;\; \text{for } a \text{ real.} \tag{13}$$

Let $\phi_t, t \in \mathbb{R}$, ($t =$ time) denote the *symplectic flow* on Γ generated by classical Hamilton fu., h, corresponding to the Hamiltonian $\widehat{H}$; see (8).

Theorem S-C: If $\widetilde{a}$ and $\widetilde{b}$ are finite (complex) measures on Γ then

$$\widehat{a} \cdot \widehat{b} - \widehat{a \cdot b} = \mathcal{O}(\varepsilon) \quad \Rightarrow \quad [\widehat{a}, \widehat{b}] = i\varepsilon \widehat{\{a, b\}} + \mathcal{O}(\varepsilon) \tag{14}$$

$$e^{i(t\widehat{H}/\varepsilon)}\, \widehat{a}\, e^{-i(t\widehat{H}/\varepsilon)} - \widehat{a \circ \phi_t} \;=\; \mathcal{O}(\varepsilon)\,. \tag{15}$$

Eq. (15) is a *Egorov-type theorem*.
For quadratic Hamiltonians (free particles, particles in const. magn. field, harmonic pot., etc.), one has that $e^{i(t\widehat{H}/\varepsilon)}\, \widehat{a}\, e^{-i(t\widehat{H}/\varepsilon)} \equiv \widehat{a \circ \phi_t}$.

3.3. Approximate position measurements

Every $\tau\,(>0)$ seconds, a pulse of light is emitted into a cavity containing the charged particle. Light scattered by the particle is caught by an array of photomultipliers that fire with pos. probability when hit by scattered photons. The firing of photomultipliers represents an *actual event* triggering a state reduction (wave-function collapse): It gives rise to a *proj. measurement* of 3 commuting "observables," $Q = (Q_1, Q_2, Q_3)$, with measured values $q = (q_1, q_2, q_3) \in \mathbb{R}^3$ corresponding to the approximate position of the charged particle. – After firing, the photo-multipliers relax back to their initial state, with a relaxation time $\ll \tau$.

Let ρ be the density matrix encoding the state of particle right *before* the firing of the photomultipliers and τ seconds *before* the next light pulse is emitted. The state, $\rho(q)$, of the particle *after* the firing of the photo-multipliers corresponding to the point $q \in \mathbb{R}^3$, but just *before* the next light pulse is emitted, is then given by

$$\rho \mapsto \rho(q) := \frac{\Phi_q^*(\rho)}{\mathrm{tr}_{\mathcal{H}}\left[\Phi_q^*(\rho)\right]}, \quad \text{with } \Phi_q^* \text{ given by}$$

$$\Phi_q^*(\rho) \equiv \Phi_{\varepsilon,q}^*(\rho) := e^{-i(\tau\widehat{H}/\varepsilon)} \sum_{\alpha} \left(\widehat{f}_{q,\alpha}\, \rho\, \widehat{f}_{q,\alpha}^*\right) e^{i(\tau\widehat{H}/\varepsilon)}, \qquad (16)$$

Approximate position measurements – ctd.

where every "amplitude" $\widehat{f}_{q,\alpha}$ is the quantization of a smooth function $f_{q,\alpha}(x)$ on $\mathbb{R}^3$ peaked at q, for all $q \in \mathbb{R}^3, \alpha = 1, 2, ..., N \ (< \infty)$, and

$$\sum_\alpha \int d\nu(q)\, f_{q,\alpha}(x)^* \cdot f_{q,\alpha}(x) \equiv 1, \quad \text{for some meas. } d\nu \text{ on } \mathbb{R}^3. \tag{17}$$

It is assumed that $|f_{q,\alpha}(x)| \approx 0$ if $|x - q| \gg \lambda$, where λ is the wave length of the light pulses scattered off the particle. (Simple models of measurements of scattered photons in photomultipliers yield (16) and (17)!)

Identity (17) implies that the map $\rho \mapsto \int d\nu(q)\, \Phi^*_q(\rho)$ is completely positive and trace-preserving.
The map $\rho \mapsto \rho(q)$ can be iterated to yield the state of the particle after $n+1$ firings of photomultipliers:

$$\rho(q_0, q_1, \ldots, q_n) = \frac{\Phi^*_{q_n} \circ \cdots \circ \Phi^*_{q_0}(\rho_0)}{\mathrm{tr}_{\mathcal{H}}\left[\Phi^*_{q_n} \circ \cdots \circ \Phi^*_{q_0}(\rho_0)\right]}, \tag{18}$$

where ρ_0 is the initial state of the particle. With the measurement data $\{q_0, q_1, \ldots, q_n\}$ of approx. particle positions we associate the density

$$\mathbb{P}^{(n)}_{\varepsilon,\rho_0}(q_0, q_1, \ldots, q_n) = \mathrm{tr}_{\mathcal{H}}\left[\Phi^*_{\varepsilon,q_n} \circ \cdots \circ \Phi^*_{\varepsilon,q_0}(\rho_0)\right], \tag{19}$$

Probabilities of position-measurement records

which is obviously **non-negative**. By (16) and (17),

$$\int \prod_{j=0}^{n} d\nu(q_j)\ \mathbb{P}^{(n)}_{\varepsilon,\rho_0}(q_0, q_1, \dots, q_n) = \mathrm{tr}_{\mathcal{H}}[\rho_0] = 1\,, \qquad (20)$$

for an arbitrary density matrix ρ_0 on $\mathcal{H}$. Thus, $\mathbb{P}^{(n)}_{\varepsilon,\rho_0}(q_0, q_1, \dots, q_n)$ can be interpreted as the *probability density on the space of position-measurements*, $\underline{q}_n := (q_0, q_1, \dots, q_n)$, conditioned on the initial state ρ_0.

We define a space, $\mathfrak{Q}$, of arbitrarily long sequences of position-measurements, $\underline{q} \equiv \underline{q}_\infty$, by

$$\mathfrak{Q} := \left(\mathbb{R}^3_q\right)^{\times \mathbb{N}_0}$$

By Kolmogorov's extension lemma the measures $\mathbb{P}^{(n)}_{\varepsilon,\rho_0}(\underline{q}_n) \prod_{j=0}^{n} d\nu(q_j)$ are the marginals of a measure $d\mathbb{P}_{\varepsilon,\rho_0}(\underline{q})$ on the space $\mathfrak{Q}$.

Remark on measurements of approximate particle positions:

The initial state, ρ_0, of the particle can be perfectly *spherically symmetric*, and the detector may also be invariant under space rotations. One may then wonder how, after a measurement, the state of the particle may be peaked near a point $0 \neq q \in \mathbb{R}^3$, thus *breaking spherical symmetry.* –

3.4. Measurement records in the semi-classical regime

The point is that the states of photons scattered off the particle are *entangled* with the state of the particle and depend on its position (operator), $\widehat{x}$. If a large number of scattered photons hitting the photomultipliers produce a measurement of the observable Q then the state of the charged particle "collapses" to one localized near a *specific point* $q \in \sigma(Q) = \mathbb{R}^3$; (a phenomenon called *"purification"* first studied by Maassen & Kümmerer; B(C)FFS, et al. $\rightarrow$ *"non-locality of QM"*.)

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Next, let ϕ_t be the symplectic flow on Γ, as in *Theorem S-C*. We set

$$\xi_j \equiv (x_j, p_j) := \phi_{j\tau}(\xi), \quad \xi \equiv (x, p) = (x_0, p_0) \in \Gamma.$$

Let $d\mu_\varepsilon(\xi)$ be the Wigner distribution of the initial state, $\rho_{\varepsilon,0}$, of the charged particle. We choose a sequence of initial states $\{\rho_{\varepsilon,0}\}_{\varepsilon>0}$ such that, in the limit as $\varepsilon \searrow 0$, $d\mu_\varepsilon(\xi)$ approaches a *positive probability measure*, $d\mu_0(\xi)$, on phase space Γ.

We now use the results stated in *Theorem S-C* to conclude that

$$\boxed{\mathbb{P}^{(n)}_{\varepsilon,\rho_{0,\varepsilon}}(q_0, q_1, \dots, q_n) = \mathbb{P}^{(n)}_{\rho_0}(q_0, q_1, \dots, q_n) + \mathcal{O}(\varepsilon),} \tag{21}$$

Semi-classical regime – ctd.

where $\mathcal{O}(\varepsilon)$ is an error term (that grows rapidly in n), and

$$\mathbb{P}^{(n)}_{\rho_0}(q_0, q_1, \ldots, q_n) = \int_\Gamma \prod_{j=0}^{n} \Big(\sum_{\alpha} \big| f_{q_j,\alpha}(x_j) \big|^2 \Big) \mathrm{d}\mu_0(\xi) \qquad (22)$$

Let

$$d\lambda(x,q) := \Big[\sum_{\alpha=1}^{N} |f_{q,\alpha}(x)|^2 \Big] d\nu(q).$$

The measures $\mathbb{P}^{(n)}_{\rho_0}(\underline{q}_n) \prod_{j=0}^{n} d\nu(q_j)$ are the marginals of a measure $d\mathbb{P}_{\mu_0}(\underline{q})$ on $\mathfrak{Q}$, given by

$$d\mathbb{P}_{\mu_0}(\underline{q}) = \int_\Gamma d\mu_0(\xi) \prod_{j=0}^{\infty} d\lambda(x_j, q_j), \qquad (23)$$

that is *"exchangeable"* (de Finetti).

One expects that *an infinitely long measurement record* $\underline{q} \in \mathrm{supp}\mathbb{P}_{\mu_0} \subset \mathfrak{Q}$ *almost surely determines a unique classical particle trajectory:*

$$\{\xi_j = (x_j, p_j)\}_{j=0}^{\infty}, \quad \text{with} \quad \xi_{j+1} = \phi_{(j+1)\tau}(\xi), \; \xi = \tilde{\xi}(\underline{q}) \in \Gamma,$$

3.5. Reconstruction of particle trajectories

where $\xi = \tilde{\xi}(\underline{q})$ is the initial condition of the classical particle trajectory singled out by $\underline{q}$.

Let Δ be a measureable subset of $\mathfrak{Q}$, and define $\Lambda(\Delta) \equiv \Lambda \in \Gamma$ by

$$\Lambda = \left\{ \xi \,\middle|\, \xi = \tilde{\xi}(\underline{q}),\ \text{for } \underline{q} \in \Delta \right\},$$

i.e., $\Lambda(\Delta)$ is the image of Δ under the map $\tilde{\xi}$. It can then be shown that

$$\mathbb{P}_{\mu_0}(\Delta) = \mu_0(\Lambda) \qquad (\textit{Born's Rule})$$

—

The construction of the map

$$\begin{aligned} \tilde{\xi} :\ & \mathfrak{Q} \to \Gamma, \\ & \underline{q} \mapsto \tilde{\xi}(\underline{q}) \in \Gamma \end{aligned} \tag{24}$$

is an "exercise" in statistics that we are only able to solve if the Hamilton function h is quadratic in x and p by using the *Law of Large Numbers* and the *Central Limit Theorem*. The example of a freely moving particle is particularly simple. We set

$$\mathfrak{p}_j := \frac{M}{\tau}\left(q_j - q_{j-1}\right).$$

The example of a freely moving particle

The *Law of Large Numbers* then implies that

$$\lim_{N\to\infty} \frac{1}{N} \sum_{j=1}^{N} \mathfrak{p}_j = p \equiv \text{momentum coo.of } \tilde{\xi}(\underline{q}).$$

By the *Central Limit Theorem*, the variable

$$\delta\mathfrak{p} := \lim_{N\to\infty} \tfrac{1}{\sqrt{N}} \textstyle\sum_{n=1}^{\infty} \Big(\mathfrak{p}_n - p\Big)$$

is a Gaussian r.v. with mean 0 and a finite variance determined by $d\lambda$, but indep. of $d\mu_0$. – Finally,

$$\frac{1}{N} \sum_{n=1}^{N} \Big(q_n - n\,\tau\,\frac{p}{M}\Big) \underset{N\to\infty}{\longrightarrow} x = \text{position coo. of } \tilde{\xi}(\underline{q})\,.$$

By (21), the probability of measnt. data $\underline{q}_n = (q_0, q_1, \ldots, q_n)$, $n < \infty$, *taken from a high-energy particle* (i.e., for ε small) *is close to the probability of* $\underline{q}_n$ *taken from a corresponding classical particle.*

$\Rightarrow$ Thus, highly energetic charged q.m. particle periodically illuminated by laser pulses leave tracks close to trajectories of classical particles.

4. Conclusions

I. It would be interesting to analyze the *quantum corrections*, present whenever $\varepsilon > 0$, to the trajectories reconstructed from the classical measures

$$\left\{\mathbb{P}^{(n)}_{\rho_0}(q_0, q_1, \dots, q_n)\right\}_{n=0,1,2,\dots}$$

defined in (22). For $\varepsilon > 0$, the sequ. $\left\{q_n\right\}_{n=0,1,2,\dots}$ is expected to exhibit diffusive noise, growing like $\sqrt{n}$, around a *classical trajectory*.

II. The quantum mechanics of an indirect measurement of the position of a charged particle by bombarding it with a large number of soft photons which then hit detectors is quite well understood within idealized models; (see, e.g., Maassen & Kümmerer, BFFS.)

III. In an earlier paper (BBFF), we have proposed and analyzed an idealized model of a charged particle periodically illuminated by laser pulses that is simple enough to be solved essentially exactly, provided the particle dynamics is quasi-free. The positions of a particle described by this model are shown to line up along Mott tracks even before the semi-classical regime is approached.

Further examples of indirect measurements

However, the exact solution of the model somewhat obscures the basic physical mechanisms giving rise to these tracks. $\longrightarrow$ Thus, we should improve our grasp of the model analyzed in this talk for $\varepsilon > 0$; (*"measurements of non-commuting observables"*!).

IV. There are plenty of examples of how, in QM, information about a physical system, *S*, is retrieved by having a long sequ. of "boring" *probes* (photons, neutrons, atoms, ...) interact with *S*, whose states get entangled with state of *S* and which are then subjected to a crude projective measurement. An instructive example is the *Haroche-Raymond experiment*. – Etc.

Thank you for your attention !

Is this the present situation of humanity?