Phase Transitions in the Tripartite Heisenberg Model

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About the model

- Spin system with tripartite, antiferromagnetic interaction.
- N particles divided into three sets A, B, C.
  Relative sizes x, y, z.
- Hamiltonian:

$$H_N = \frac{1}{N}\Big(\sum_{\substack{i\in A \\ j\in B\cup C}} \vec{S}_i \cdot \vec{S}_j + \sum_{\substack{i\in B \\ j\in C\cup A}} \vec{S}_i \cdot \vec{S}_j + \sum_{\substack{i\in C \\ j\in A\cup B}} \vec{S}_i \cdot \vec{S}_j \Big). \qquad (1)$$

  where $\vec{S}_i$ is the spin operator of particle i.

Equilibrium in thermodynamic limit

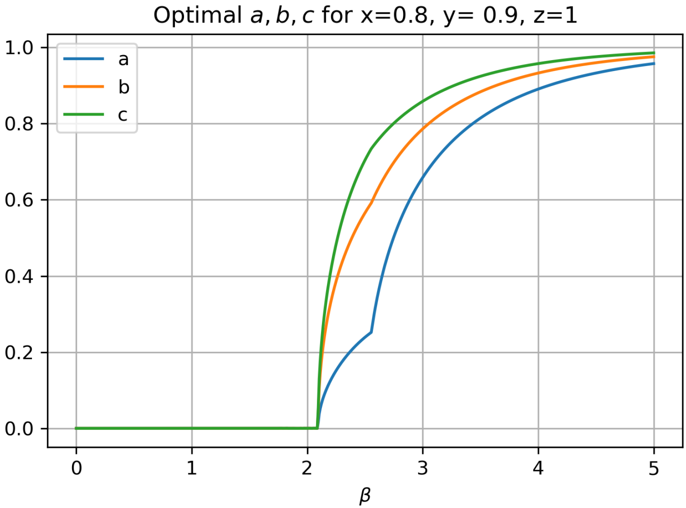
- Macroscopic state is magnetisation vectors $\vec{a}, \vec{b}, \vec{c} \in \mathbb{R}^3$.
  Magnitude $0 \leq a, b, c \leq 1$.
- Minimise free energy:

$$\Gamma = \max_{\vec{a},\vec{b},\vec{c}} (S - \beta E) =$$

$$= \max_{\vec{a},\vec{b},\vec{c}} \left[xf(a) + yf(b) + zf(c) - \frac{\beta}{2} \left(x\vec{a} \cdot y\vec{b} + y\vec{b} \cdot z\vec{c} + z\vec{c} \cdot x\vec{a} \right) \right]$$

where $f(t) = (1 - t) \tanh^{-1}(t) - \ln(1 + t)$.

Argmax as function of temperature



Results for optimal a, b, c as function of β

- are 0 for $\beta \leq \frac{2}{x}, \frac{2}{y}, \frac{2}{z}$, and positive for $\beta \geq \frac{2}{x}, \frac{2}{y}, \frac{2}{z}$.
- Phase transition at the smallest positive root β_1 to the following cubic polynomial:

$$xyz\beta^3 - (xy + yz + zx)\beta^2 + 4 = 0 \tag{2}$$

- positive precisely when $\beta > \beta_1$ (but exception $x < y = z$).