Finite Speed of Boson Transport in the Presence of Long-Range Interactions

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joint work with Marius Lemm and Jingxuan Zhang

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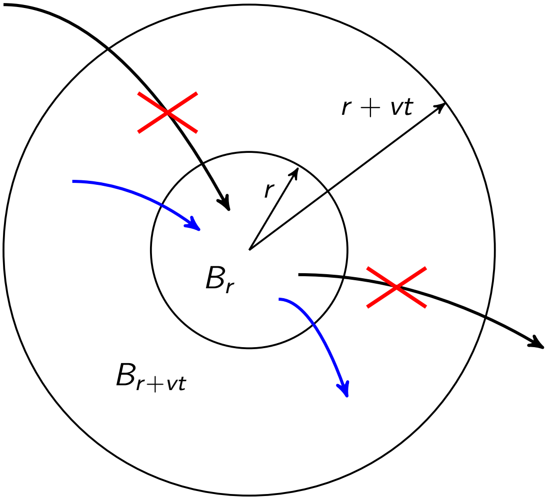
Goal



Set-up

Consider a finite lattice $\Lambda \in \mathbb{R}^d$ and Hamiltonian of the form

$$H = \sum_{x,y\in\Lambda} J_{xy} b_x^\dagger b_y + V$$

- $b_x, b_x^\dagger$ are the creation and annihilation operators at site x
- $V = \Phi(\{n_x\}_{x\in\Lambda})$ with Φ arbitrary and $n_x = b_x^\dagger b_x$
- Example of hopping matrix $J_{x,y}$

  Long range: $|J_{xy}| \leq C|x-y|^{-\alpha}$

  Finite range: $J_{xy} = 0$ if $|x-y| > \rho$

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Notation: $N_X = \sum_{x \in X} n_x$ number op. on a region $X \subset \Lambda$

Main result: Assumptions

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$$\exists \, \lambda_2 > \lambda_1 > 0 \quad \text{s.t.} \quad \lambda_1 \leq \langle n_x \rangle_0 \leq \lambda_2 \quad \forall x \in \Lambda$$

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An important role is played by the first moment of the hopping matrix

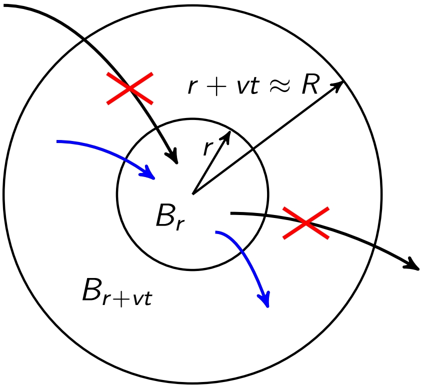
$$\kappa = \sup_{x \in \Lambda} \sum_{y \in \Lambda} |J_{xy}| \, |x-y| < \infty$$

Main Result

For every $v > 12\kappa$ and for all R, $r \geq 1$ with $R - r$ big enough and initial states $\psi_0 \in \mathcal{D}(N_\Lambda^{1/2})$ with controlled density, it holds for every $vt \leq R - r$

$$(UB) \qquad \langle N_{B_r} \rangle_t \leq \langle N_{B_R} \rangle_0 \, e^{vt/(R-r)}$$

$$(LB) \qquad \langle N_{B_R} \rangle_t \geq \langle N_{B_r} \rangle_0 \, e^{-vt/(R-r)}$$



Bosons move at most with speed v, for all $v > 12\kappa$

$\Rightarrow$ 12κ bounds the maximal speed of propagation

Remarks

We can extend this to higher moments of the number operator, with requiring the same decay of the interactions.

interesting because it can help us refine Lieb-Robinson bounds for long range bosons

- ☺ Valid for long range interactions
- ☺ "Thermodynamically stable"

  i.e. no global number operator appearing in the error term
- ☹ Assumptions on the initial state, still physically relevant

Tools

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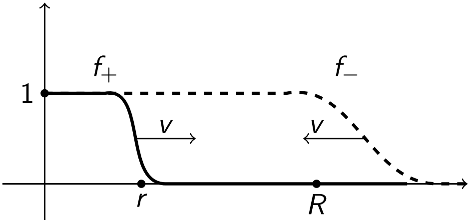
2) **Downward multi-scale induction**

ASTLO

Given a special smooth cut-off function f, define the ASTLOs as

$$N_{f_\pm} := \sum_{x \in \Lambda} n_x f_\pm(x)$$



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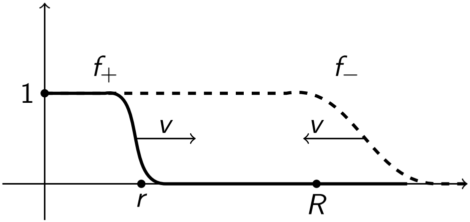

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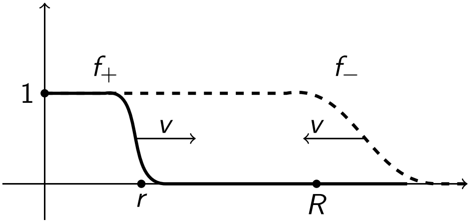

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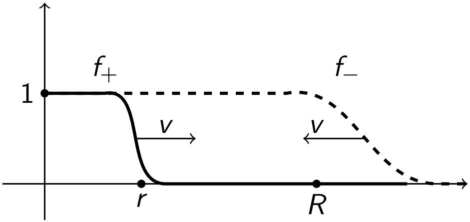
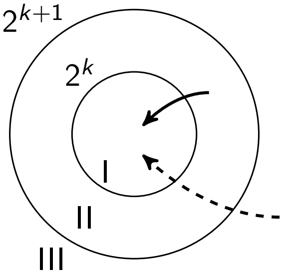


- Track particles on the region of interest in a smooth way
- Their time evolution can be controlled
- Can be related to the number operator on the region of interest

Multi scale induction

- At each induction step we control the particles moving from region II to region I
- The induction hypothesis has already accounted for the contribution from region III into region I



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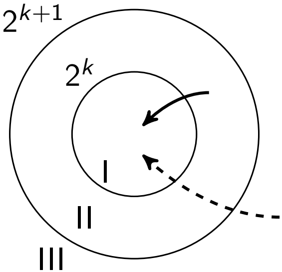

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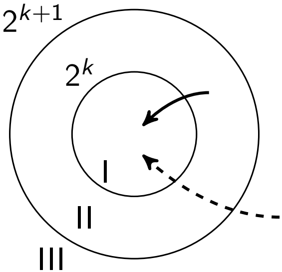

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  - Allows to control particles far away
  - Makes the bound “thermodynamically stable”

Summary and Future Directions

Summary:

- Under the assumption of bounded density we controlled particle propagation for polynomially decaying interactions
- First tool: ASTLO

    Tracks particles across the lattice

    Their time evolution can be controlled
- Second tool: Downwards multi-scale induction

    To control the contribution from particles far away

Future Directions:

- Refine thermodyn. stable long range Lieb-Robinson bounds for bosonic system
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Thank You For Your Attention!