Validity of edge linear response for 2d topological insulators

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Introduction

1. Justifying employment of Kubo's formula for quantum transport

$$\mathcal{H}(t) = \mathcal{H}_0 + e^{\eta t}\mathcal{P} \qquad \langle \mathcal{O} \rangle(t) \approx \langle \mathcal{O} \rangle_0 - i \int_{-\infty}^{t} e^{\eta s} \langle [\tau_t(\mathcal{O}), \tau_s(\mathcal{P})] \rangle_0 \, \mathrm{d}s$$

   for gapless $\mathcal{H}_0$ and selected $\mathcal{O}$ [Simon '84: Problem 4.B];

2. In the setting of the quantum Hall effect, express edge conductance σ_E and other edge physical quantities in terms of dynamical response functions;
   - $\sigma_E = \sigma_B := \sigma_{12}$ by bulk-edge correspondence ([Hatsugai '93; Schulz-Baldes et al.; Graf et al.; Fröhlich et al.]);
   - linear response theory for σ_B verified in adiabatic limit: reliant on presence of spectral gap in bulk [Avron-Seiler-Yaffe '87, Bachmann-Bols-De Roeck-Fraas '18];
   - Edge Hamiltonian is gapless.

Aim: in a suitable scaling limit, to show that the validity of edge linear response and the quantization of the edge conductance can be derived from first principles, from the time-dependent Schrödinger equation.

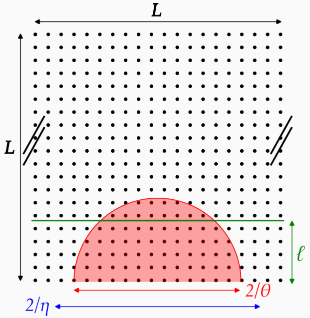
Setting (I)

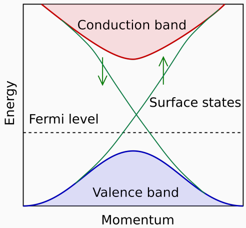
Non-interacting fermions on lattice $\Lambda_L = \Gamma_L \times S_M$ on cylinder:

$$\mathcal{H}(\eta t) = \sum_{\mathbf{x},\mathbf{y}\in\Lambda_L} a^*_{\mathbf{x}} \left[H_L(\mathbf{x};\mathbf{y}) - e^{\eta t}\theta\mu_L(\theta x)\delta_{\mathbf{x},\mathbf{y}} \right] a_{\mathbf{y}} \qquad \eta,\theta \ll 1, t \leq 0$$

$H_L \in \mathcal{B}(\ell^2(\Lambda_L))$ horizontally translation-invariant, finite-ranged, and presenting gap-closing edge states exponentially localised near $\partial\Gamma_L$;
$\mu_L \in \mathcal{C}^\infty_c(S^1_L \times \mathbb{R})$ bump function at lower edge $\partial_+\Gamma_L$;

Observables: $x \in \partial_+\Gamma_L$, integrated 2-current $j^\ell_{\nu,x} = \sum_{k\leq\ell} j_{\nu,x+ke_2}$





Setting (II)

Dynamics :
$$\begin{cases} i\partial_t \rho(t) = [\mathcal{H}(\eta t), \rho(t)] \\ \rho(-\infty) = \rho_{\beta,L} = e^{-\beta(\mathcal{H}(-\infty)-\mu_0\mathcal{N})}/Z_{\beta,L,\mu_0} \end{cases}$$

Response :
$$\chi_\nu^\ell(x,\theta,\eta) := \lim_{\beta,L\to\infty} \frac{\operatorname{tr} j_{\nu,x}^\ell \rho(0) - \operatorname{tr} j_{\nu,x}^\ell \rho_{\beta,L}}{\theta}$$

Interested in its behaviour for $\ell \to \infty$ after $\eta, \theta \to 0$ limits.

Duhamel expansion

$$\operatorname{tr} j_{\nu,x}^\ell(\rho(0) - \rho_{\beta,L}) = \sum_{n=1}^{\infty} (-i)^n \int_{-\infty\le s_n \le \ldots \le s_1 \le 0} d\underline{s}\, e^{\eta(s_1+\ldots+s_n)} \cdot \langle [\cdots[[j_{\nu,x}^\ell, \tau_{s_1}(\mathcal{P})], \tau_{s_2}(\mathcal{P})]\cdots, \tau_{s_n}(\mathcal{P})]\rangle_{\beta,L}.$$

- difficult control of real time dynamics;
- to prove validity of linear response, we need bounds summable in n and small in η, θ for $n \ge 2$;
- gaplessness $\Longrightarrow$ not a power series in θ.

Result

Theorem [M. Porta, H.P. S.; in preparation]

For $\theta = a\eta$ with $a \leq C\,|\log \eta|$, $\forall \eta < \eta_0$ the expansion is convergent, and

$$\chi^{\ell}_{\nu}(x;\theta,\eta) = \sum_{\omega}^{*} \frac{v^{\nu}_{\omega}}{2\pi|v_{\omega}|} \int_{\mathbb{R}} \hat{\mu}(q;0) e^{iq\theta x} \frac{v_{\omega} q}{-i/a + v_{\omega} q} \frac{\mathrm{d}q}{2\pi} + O(\ell\eta^{\gamma} + e^{-c\ell})$$

where $v^{\nu}_{\omega} = \delta_{\nu,0} + v_{\omega}\delta_{\nu,1}$, and summation is over modes localised at the edge. In particular, leading order is

$$\chi^{\ell}_{\nu}(x;\theta,\eta) = o(1) \qquad \text{if } a \ll 1$$

$$\chi^{\ell}_{\nu}(x;\theta,\eta) = \sum_{\omega}^{*} \frac{v^{\nu}_{\omega}}{2\pi|v_{\omega}|} \mu(\theta x) \qquad \text{if } a \gg 1$$

For $\nu = 1$ we recover the edge conductance $\chi^{\ell}_{\nu}(x;\theta,\eta) = \sum^{*}_{\omega} \frac{\operatorname{sgn} v_{\omega}}{2\pi} \mu(\theta x)$.

Response is non-trivial also for $a = O(1)$ (possibly related to generalized hydrodynamic models? [Doyon '20] notes on GHD).

Comments and future perspectives

Main ingredients of proof:

a) Duhamel exp. represented in terms of imaginary time-ordered cumulants [Greenblatt-Lange-Marcelli-Porta '24];
b) Loop cancellation: said cumulants given by sum of 1-loop Feynman diagrams. $O(1)$ contributions to the sum, present due to absence of spectral gap, cancel out; related to bosonization in $1+1d$ relativistic theories (see *e.g.* [Fröhlich-Gotschmann-Marchetti '95]);
c) After cancellation of the scaling limit contribution, extraction of summable bounds for higher order terms in the Duhamel expansion.

Future perspectives:

* Extension to the interacting case, via RG analysis and Ward identities: work in progress w/ M. Porta & G. Scola;
* Extension to weakly disordered systems;
* Extension to higher dimensional cases, e.g. 3d Weyl semimetals?

Thank you for your kind attention!