Bulk Hall Conductivity for Periodic Interacting Fermions

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Quantissima V

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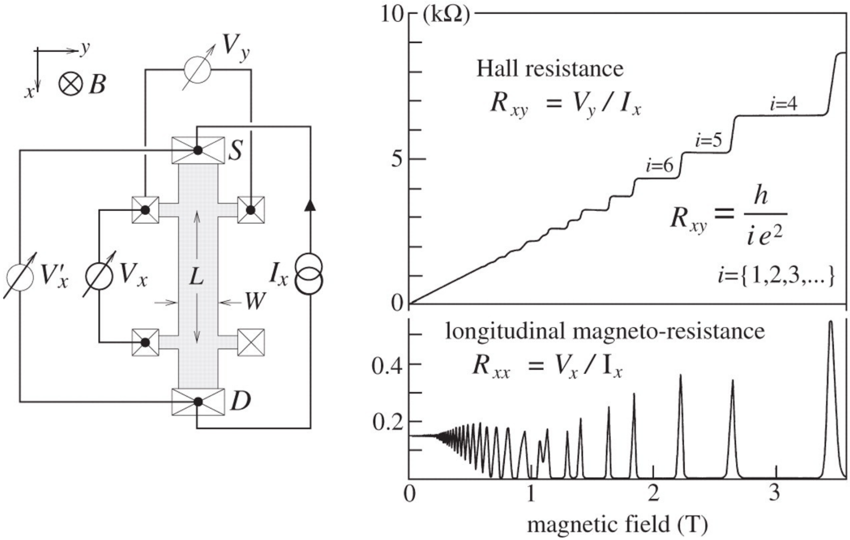
The Quantum Hall Effect



Setup

- One-particle Hilbert space $\mathscr{H} := \ell^2(\mathbb{Z}^2, \mathbb{C}^n)$
- Associated CAR-algebra $\mathcal{A} := \mathrm{CAR}(\mathscr{H}) \subseteq \mathcal{B}(\mathcal{F}_{(-)}(\mathscr{H}))$
- States are described as normalised positive linear functionals $\omega \colon \mathcal{A} \to \mathbb{C}$
- Hamiltonian H of the system is given by an SLT-operator

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Model and Assumptions

- We assume that the Hamiltonian H

  (i) is short-range

  (ii) has a unique gapped ground state ω_0

  (iii) is translation invariant

- (i) + (ii) $\implies$ adiabatically switching on a linear potential $H \longrightarrow H + \varepsilon X_1$, the system evolves from the ground state ω_0 to a NEASS ω_ε

- (iii) $\implies$ ω_0, ω_ε and $J_k = \mathrm{i}[H, X_k]$ are translation invariant

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Result

- It holds that

$$\overline{\omega_\varepsilon}(\mathrm{i}[H, X_2]) = \varepsilon\,\sigma_{\mathrm{H}} + \mathcal{O}(\varepsilon^\infty) \quad \text{with} \quad \sigma_{\mathrm{H}} = -\mathrm{i}\overline{\omega_0}([X_1^{\mathrm{OD}}, X_2^{\mathrm{OD}}])$$

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$$\overline{\omega_\varepsilon}(\mathrm{i}[H, X_1]) = \mathcal{O}(\varepsilon^\infty)$$

- σ_{H} is a function of ω_0
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- In invertible phases $\sigma_{\mathrm{H}} \in \frac{1}{2\pi}\mathbb{Z}$

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Thank you for your Attention!

Definition

A translation is a map $T : \mathbb{Z}^d \to \mathrm{Aut}(\mathcal{A})$ *that satisfies the following properties:*

(i) *For all* $\gamma \in \mathbb{Z}^d$ *and* $M \subseteq \mathbb{Z}^d$ *it holds that* $T_\gamma(\mathcal{A}_M) = \mathcal{A}_{M+\gamma}$.

(ii) *For all* $\gamma \in \mathbb{Z}^d$ *and* $x \in \mathbb{Z}^d$, *it holds that* $T_\gamma n_x = n_{(x+\gamma)}$.

We say a state ω is T-periodic, if

$$\forall \gamma \in \mathbb{Z}^d, A \in \mathcal{A} : \quad \omega(T_\gamma A) = \omega(A)\,.$$

An interaction Φ is called T-periodic, if

$$\forall \gamma \in \mathbb{Z}^d, M \in P_0(\mathbb{Z}^d) : \quad T_\gamma \Phi(M) = \Phi(M + \gamma)\,.$$

Definition

Let Φ be an interaction and $d \in \mathbb{N}$. Let

$$\|\Phi\|_d := \sup_{x\in\mathbb{Z}^2} \sum_{\substack{M\in P_0(\mathbb{Z}^2)\\ x\in M}} (1+\operatorname{diam}(M))^d \|\Phi(M)\| \ .$$

The set of all interactions with finite $\|\cdot\|_d$ is denoted by B_d. We also define $B_\infty := \bigcap_{d\in\mathbb{N}_0} B_d$. For $a > 0$ let

$$\|\Phi\|_{\exp,a} := \sup_{x\in\mathbb{Z}^2} \sum_{\substack{M\in P_0(\mathbb{Z}^2)\\ x\in M}} \exp(a \operatorname{diam}(M))\|\Phi(M)\|$$

and denote the set of all interactions with finite $\|\cdot\|_{\exp,a}$ by $B_{\exp,a}$.

Definition

For two interactions Φ and Ψ their commutator is given by

$$[\Phi, \Psi] : P_0(\mathbb{Z}^2) \to \mathcal{A}^N , \qquad M \mapsto [\Phi, \Psi](M) \ := \sum_{\substack{M_1, M_2 \subset M \\ M_1 \cup M_2 = M}} [\Phi(M_1), \Psi(M_2)]$$

and $\mathrm{i}[\Phi, \Psi]$ *is again an interaction.*

Definition

Given an automorphism α, *such that* $\alpha D_\infty^N = D_\infty^N$ *and* $\alpha T_\gamma = T_\gamma \alpha$ *for all* $\gamma \in \mathbb{Z}^2$ *and* $j \in \{1, 2\}$, *we define the quasi-local observable*

$$(X_j^{\mathrm{OD}\alpha})_* := \mathrm{i}\alpha^{-1} \int_{\mathbb{R}} \mathrm{d}s \; W_g(s)\, \mathrm{e}^{\mathrm{i}s\mathcal{L}_H} \alpha \mathcal{L}_{X_j} \alpha^{-1} H_0$$

and the map $X_j^{\mathrm{OD}\alpha} \colon P_0(\mathbb{Z}^2) \to \mathcal{A}^N$ *such that for* $k \in \mathbb{N}$, $\gamma \in \mathbb{Z}^2$

$$X_j^{\mathrm{OD}\alpha}(\Lambda_k + \gamma) = T_\gamma(\mathbb{E}_{\Lambda_k}(X_j^{\mathrm{OD}\alpha})_* - \mathbb{E}_{\Lambda_{k-1}}(X_j^{\mathrm{OD}\alpha})_*)$$
$$X_j^{\mathrm{OD}\alpha}(\Lambda_0 + \gamma) = T_\gamma \mathbb{E}_{\Lambda_0}(X_j^{\mathrm{OD}\alpha})_*$$

and $X_j^{\mathrm{OD}\alpha}(M) = 0$ *if there are no* $k \in \mathbb{N}_0$, $\gamma \in \mathbb{Z}^2$ *such that* $M = \Lambda_k + \gamma$.