EBERHARD KARLS UNIVERSITÄT TÜBINGEN

From decay of correlations to stability and locality of the Gibbs state

Tom Wessel (joint work with Capel, Moscolari, Teufel)

13. August 2024 at Quantissima

FOR 5413

CRC TRR 352

Mathematical Setup for Spin Lattice Systems

- Finite ν-dimensional lattice $\Lambda \subset \mathbb{Z}^\nu$ with ℓ^1-distance d
- Hilbert space $\mathcal{H}_Z = \bigotimes_{z \in Z} \mathcal{H}_z$ with local Hilbert space $\mathcal{H}_z = \mathbb{C}^D$
- Operator algebra $\mathcal{A}_Z = \mathcal{B}(\mathcal{H}_Z)$ describing operators localized in $Z \subset \Lambda$ with identification $A \in \mathcal{A}_Z \leftrightarrow A \otimes 1_{\Lambda \setminus Z} \in \mathcal{A}_\Lambda$
- local Hamiltonian $H_\Lambda = \sum_{Z \subset \Lambda} \Psi(Z)$ where $\|\Psi(Z)\|$ decays in size of Z.
- Gibbs state $\rho^\Lambda[H_\Lambda] = \frac{e^{-\beta H_\Lambda}}{\mathrm{tr}(e^{-\beta H_\Lambda})}$

Mathematical Setup for Spin Lattice Systems

- Finite ν-dimensional lattice $\Lambda \subset \mathbb{Z}^\nu$ with ℓ^1-distance d
- Hilbert space $\mathcal{H}_Z = \bigotimes_{z \in Z} \mathcal{H}_z$ with local Hilbert space $\mathcal{H}_z = \mathbb{C}^D$
- Operator algebra $\mathcal{A}_Z = \mathcal{B}(\mathcal{H}_Z)$ describing operators localized in $Z \subset \Lambda$ with identification $A \in \mathcal{A}_Z \leftrightarrow A \otimes 1_{\Lambda \setminus Z} \in \mathcal{A}_\Lambda$
- local Hamiltonian $H_\Lambda = \sum_{Z \subset \Lambda} \Psi(Z)$ where $\|\Psi(Z)\|$ decays in size of Z.
- Gibbs state $\rho^\Lambda[H_\Lambda] = \dfrac{e^{-\beta H_\Lambda}}{\mathrm{tr}(e^{-\beta H_\Lambda})}$

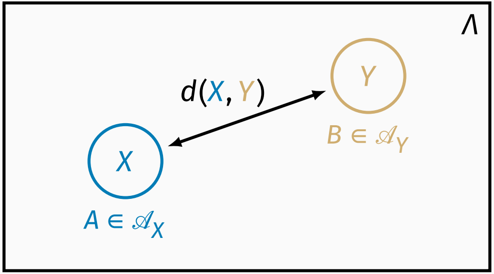
Goal: Understand locality of the Gibbs state $\rho^\Lambda[H_\Lambda]$

Different notions of locality

Decay of Correlations

$$\left|\mathrm{Cov}_\rho(A, B)\right| := \left|\mathrm{tr}\big(\rho\, A\, B\big) - \mathrm{tr}\big(\rho\, A\big)\, \mathrm{tr}\big(\rho\, B\big)\right|$$
$$\leq C\, \|A\|\, \|B\|\, |X|\, |Y|\, \mathrm{e}^{-c d(X,Y)}$$

for all $X, Y \subset \Lambda$, $A \in \mathscr{A}_X$, $B \in \mathscr{A}_Y$

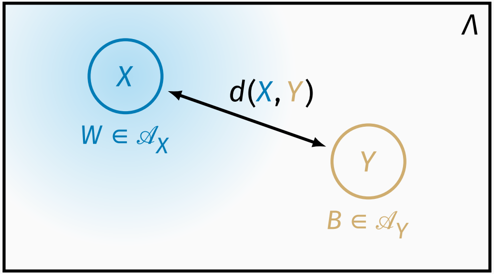


Different notions of locality

Local perturbations perturb locally (LPPL)

$$\left|\operatorname{tr}\big(\rho[H]\,B\big) - \operatorname{tr}\big(\rho[H+W]\,B\big)\right| \le C\, e^{3\beta\|W\|}\, \|B\|\, |X|\, |Y|\, e^{-c d(X,Y)}$$

for all $X, Y \subset \Lambda$, $W \in \mathscr{A}_X$, $B \in \mathscr{A}_Y$

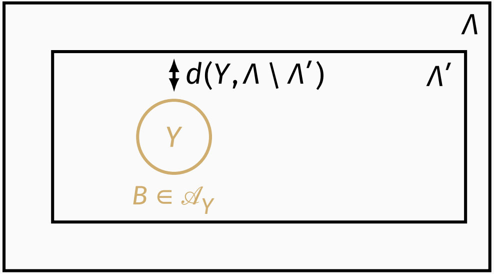


Different notions of locality

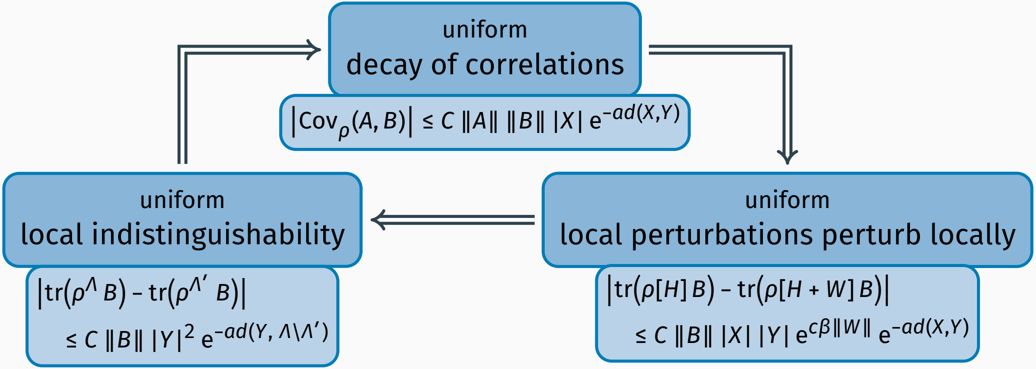
Local indistinguishability

$$\left|\mathrm{tr}\left(\rho^{\Lambda} B\right) - \mathrm{tr}\left(\rho^{\Lambda'} B\right)\right| \le C \, \|B\| \, |Y|^2 \, e^{-c d(Y, \Lambda \setminus \Lambda')}.$$

for all $Y \subset \Lambda' \subset \Lambda$, $B \in \mathscr{A}_Y$



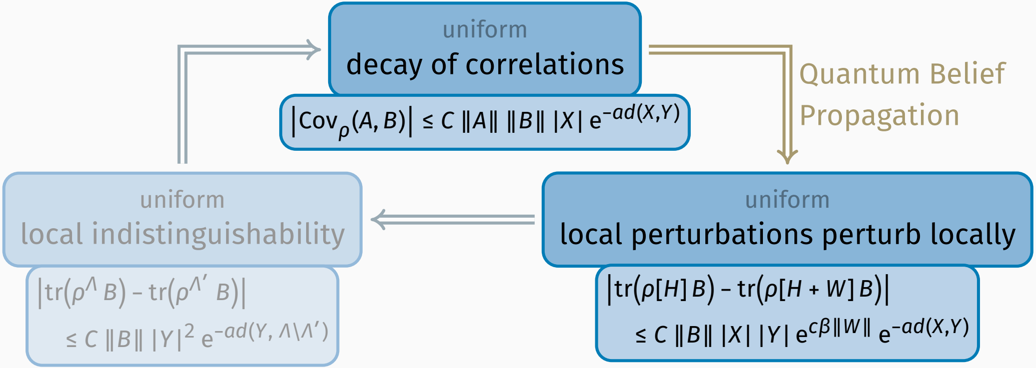
Result


uniform
decay of correlations
$\left|\mathrm{Cov}_\rho(A, B)\right| \le C\, \|A\|\, \|B\|\, |X|\, \mathrm{e}^{-ad(X,Y)}$
uniform
local indistinguishability
$\left|\mathrm{tr}\left(\rho^{\Lambda} B\right) - \mathrm{tr}\left(\rho^{\Lambda'} B\right)\right| \le C\, \|B\|\, |Y|^2\, \mathrm{e}^{-ad(Y,\, \Lambda \setminus \Lambda')}$
uniform
local perturbations perturb locally
$\left|\mathrm{tr}\left(\rho[H]\, B\right) - \mathrm{tr}\left(\rho[H+W]\, B\right)\right| \le C\, \|B\|\, |X|\, |Y|\, \mathrm{e}^{c\beta\|W\|}\, \mathrm{e}^{-ad(X,Y)}$

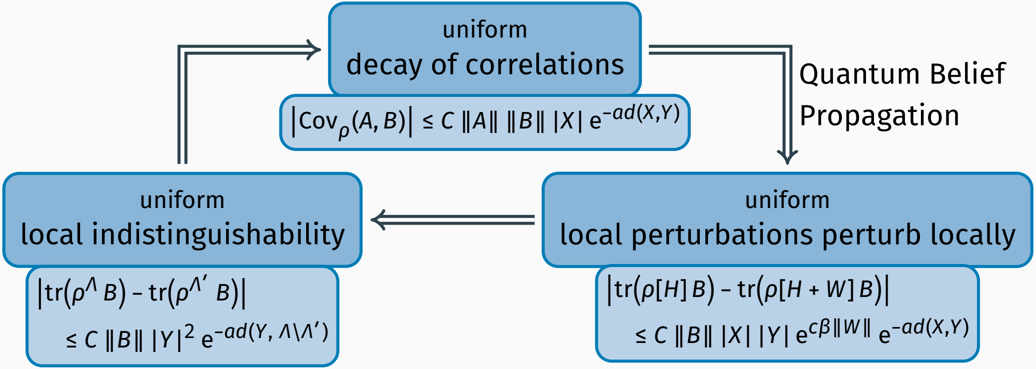
Interpolation $H(s) = H + s\,W$

ODE for Gibbs state $\frac{\mathrm{d}}{\mathrm{d}s}\rho(s) = -\frac{\beta}{2}\left\{\rho(s), \Phi_\beta^{H(s)}\left(W - \langle W\rangle_{\rho(s)}\right)\right\}$ where $\Phi_\beta^{H(s)}$ is local

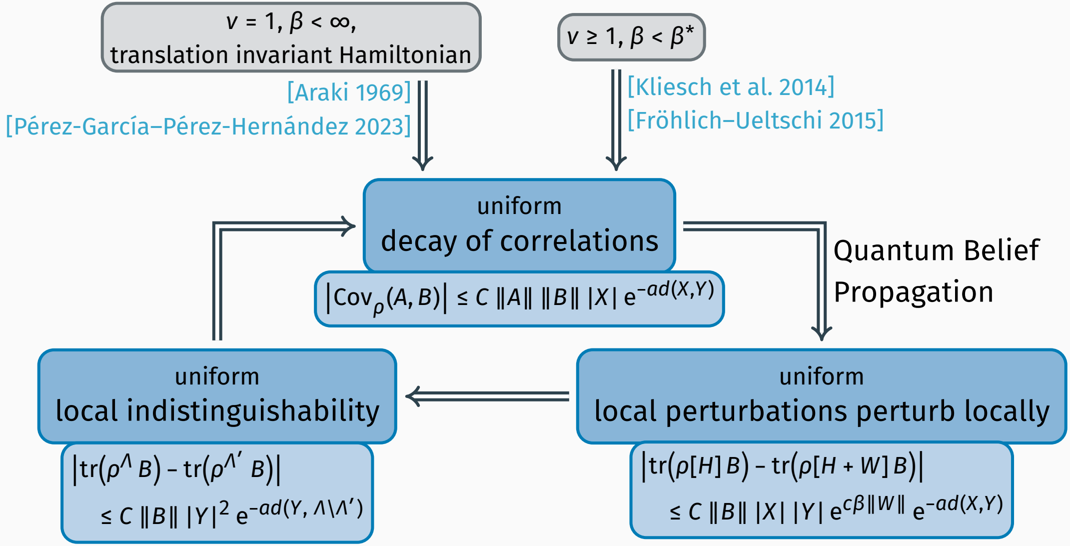
Originally introduced by [Hastings 2007]



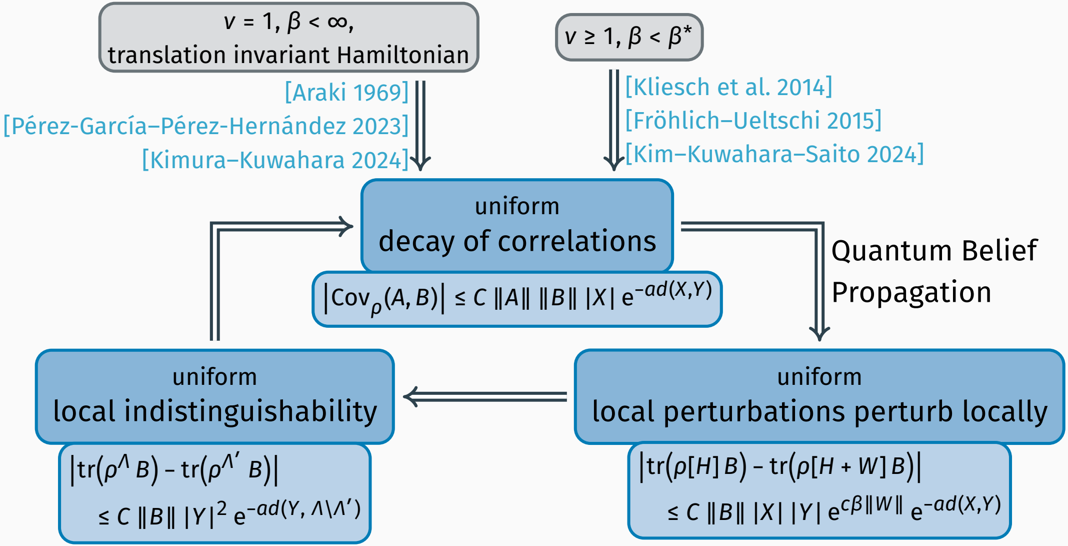
Result


uniform
decay of correlations
$\left|\mathrm{Cov}_\rho(A, B)\right| \leq C \, \|A\| \, \|B\| \, |X| \, \mathrm{e}^{-ad(X,Y)}$
Quantum Belief Propagation
uniform
local perturbations perturb locally
$\left|\mathrm{tr}\big(\rho[H]\,B\big) - \mathrm{tr}\big(\rho[H+W]\,B\big)\right|$
$\leq C \, \|B\| \, |X| \, |Y| \, \mathrm{e}^{c\beta\|W\|} \, \mathrm{e}^{-ad(X,Y)}$
uniform
local indistinguishability
$\left|\mathrm{tr}\big(\rho^{\Lambda} B\big) - \mathrm{tr}\big(\rho^{\Lambda'} B\big)\right|$
$\leq C \, \|B\| \, |Y|^2 \, \mathrm{e}^{-ad(Y,\, \Lambda \setminus \Lambda')}$

Result


$\nu = 1, \beta < \infty$,
translation invariant Hamiltonian
$\nu \geq 1, \beta < \beta^*$
[Araki 1969]
[Pérez-García–Pérez-Hernández 2023]
[Kliesch et al. 2014]
[Fröhlich–Ueltschi 2015]
uniform
decay of correlations
$\left|\mathrm{Cov}_\rho(A, B)\right| \leq C \, \|A\| \, \|B\| \, |X| \, e^{-a d(X,Y)}$
Quantum Belief
Propagation
uniform
local indistinguishability
$\left|\mathrm{tr}(\rho^\Lambda B) - \mathrm{tr}(\rho^{\Lambda'} B)\right|$
$\leq C \, \|B\| \, |Y|^2 \, e^{-a d(Y, \Lambda \setminus \Lambda')}$
uniform
local perturbations perturb locally
$\left|\mathrm{tr}\big(\rho[H] B\big) - \mathrm{tr}\big(\rho[H + W] B\big)\right|$
$\leq C \, \|B\| \, |X| \, |Y| \, e^{c\beta \|W\|} \, e^{-a d(X,Y)}$

Result

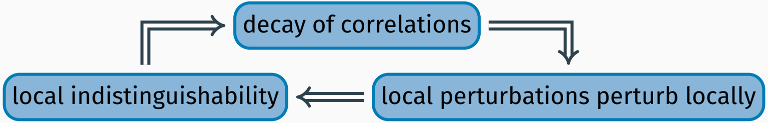

$\nu = 1, \beta < \infty$,
translation invariant Hamiltonian
[Araki 1969]
[Pérez-García–Pérez-Hernández 2023]
[Kimura–Kuwahara 2024]
$\nu \geq 1, \beta < \beta^*$
[Kliesch et al. 2014]
[Fröhlich–Ueltschi 2015]
[Kim–Kuwahara–Saito 2024]
uniform
decay of correlations
$\left|\mathrm{Cov}_\rho(A,B)\right| \leq C\,\|A\|\,\|B\|\,|X|\,\mathrm{e}^{-ad(X,Y)}$
Quantum Belief Propagation
uniform
local indistinguishability
$\left|\mathrm{tr}\left(\rho^{\Lambda} B\right) - \mathrm{tr}\left(\rho^{\Lambda'} B\right)\right|$
$\leq C\,\|B\|\,|Y|^2\,\mathrm{e}^{-ad(Y,\,\Lambda\setminus\Lambda')}$
uniform
local perturbations perturb locally
$\left|\mathrm{tr}\left(\rho[H]\,B\right) - \mathrm{tr}\left(\rho[H+W]\,B\right)\right|$
$\leq C\,\|B\|\,|X|\,|Y|\,\mathrm{e}^{c\beta\|W\|}\,\mathrm{e}^{-ad(X,Y)}$

Summary and Outlook

arXiv:2310.09182

Summary

- Rigorous proof of implications for short-range interactions with general assumptions on exact decay
- (In principle) also works for long-range interactions and lattice fermions



Open questions

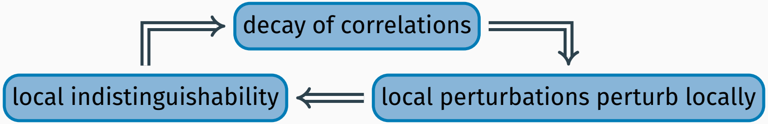
- Understand small temperatures and the zero temperature limit
- Understand the situation in open quantum systems

Summary and Outlook

arXiv:2310.09182

Summary

- Rigorous proof of implications for short-range interactions with general assumptions on exact decay
- (In principle) also works for long-range interactions and lattice fermions



Open questions

- Understand small temperatures and the zero temperature limit
- Understand the situation in open quantum systems

Thank you for your attention!

References i

- Araki, Huzihiro (1st June 1969). **'Gibbs states of a one dimensional quantum lattice'.** In: *Communications in Mathematical Physics* 14.2, pp. 120–157. DOI: 10.1007/BF01645134.
- Bachmann, Sven, Wojciech de Roeck et al. (8th Sept. 2022). **'Stability of invertible, frustration-free ground states against large perturbations'.** In: *Quantum* 6, 793. ISSN: 2521-327X. DOI: 10.22331/q-2022-09-08-793. arXiv: 2110.11194 [math-ph].
- Bachmann, Sven, Spyridon Michalakis et al. (Feb. 2012). **'Automorphic Equivalence within Gapped Phases of Quantum Lattice Systems'.** In: *Communications in Mathematical Physics* 309.3, pp. 835–871. DOI: 10.1007/s00220-011-1380-0. arXiv: 1102.0842 [math-ph].

References ii

- Fröhlich, Jürg and Daniel Ueltschi (May 2015). **'Some properties of correlations of quantum lattice systems in thermal equilibrium'.** In: *Journal of Mathematical Physics* 56.5, 053302. ISSN: 0022-2488. DOI: 10.1063/1.4921305. arXiv: 1412.2534 [math-ph].
- Hastings, Matthew B. (Nov. 2007). **'Quantum belief propagation: An algorithm for thermal quantum systems'.** In: *Physical Review B* 76.20, 201102. DOI: 10.1103/PhysRevB.76.201102. arXiv: 0706.4094.
- Henheik, Joscha, Stefan Teufel and Tom Wessel (18th Jan. 2022). **'Local stability of ground states in locally gapped and weakly interacting quantum spin systems'.** In: *Letters in Mathematical Physics* 112.1, 9. ISSN: 1573-0530. DOI: 10.1007/s11005-021-01494-y. arXiv: 2106.13780 [math-ph].

- Kim, Donghoon, Tomotaka Kuwahara and Keiji Saito (Apr. 2024). **'Thermal Area Law in Long-Range Interacting Systems'.** arXiv: 2404.04172 [quant-ph].
- Kimura, Yusuke and Tomotaka Kuwahara (18th Mar. 2024). **'Clustering theorem in 1D long-range interacting systems at arbitrary temperatures'.** arXiv: 2403.11431 [quant-ph].
- Kliesch, Martin et al. (July 2014). **'Locality of Temperature'.** In: *Physical Review X* 4.3, 031019. DOI: 10.1103/PhysRevX.4.031019. arXiv: 1309.0816.
- Pérez-García, David and Antonio Pérez-Hernández (3rd Jan. 2023). **'Locality Estimates for Complex Time Evolution in 1D'.** In: *Communications in Mathematical Physics* 399, pp. 929–970. DOI: 10.1007/s00220-022-04573-w. arXiv: 2004.10516.